

$F(\mathcal{R})$ Supergravity and Early Universe: the Meeting Point of Cosmology and High-Energy Physics

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1. Introduction

In this Chapter we focus on the field-theoretical description of the inflationary phase of the early universe and its post-inflationary dynamics (reheating and particle production) in the context of supergravity, based on the original papers (1–10). To begin with, let us first introduce some basics of inflation.

Cosmological inflation (a phase of ‘rapid’ quasi-exponential accelerated expansion of universe) (11–13) predicts homogeneity of our Universe at large scales, its spatial flatness, large size and entropy, and the almost scale-invariant spectrum of cosmological perturbations, in good agreement with the WMAP measurements of the CMB radiation spectrum (14; 15). Inflation is also the only known way to generate structure formation in the universe via amplifying quantum fluctuations in vacuum.

However, inflation is just the cosmological paradigm, not a theory! The known field-theoretical mechanisms of inflation use a slow-roll scalar field ϕ (called inflaton) with proper scalar potential $V(\phi)$ (12; 13).

The scale of inflation is well beyond the electro-weak scale, ie. is well beyond the Standard Model of Elementary Particles! Thus the inflationary stage in the early universe is the most powerful High-Energy Physics (HEP) accelerator in Nature (up to $10^{10} TeV$). Therefore, inflation is the great and unique window to HEP!

The nature of inflaton and the origin of its scalar potential are the big mysteries.

Throughout the paper the units $\hbar = c = 1$ and the spacetime signature $(+, -, -, -)$ are used. See ref. (16) for our use of Riemann geometry of a curved spacetime.

The Cosmic Microwave Background (CMB) radiation from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite mission (14) is one of the main sources of data about the early universe. Deciphering the CMB in terms of the density perturbations, gravity wave polarization, power spectrum and its various indices is a formidable task. It also requires the heavy CMB mathematical formalism based on General Relativity — see eg., the textbooks (17–19). Fortunately, we do not need that formalism for our purposes, since the relevant indices can also be introduced in terms of the inflaton scalar potential (Sec. 4). We assume

that inflation did happen. There exist many inflationary models — see eg. the textbook (13) for their description and comparison (without supersymmetry). Our aim is a viable theoretical description of inflation in the context of supergravity.

The main Cosmological Principle of a spatially homogeneous and isotropic (1 + 3)-dimensional universe (at large scales) gives rise to the FLRW metric

$$ds_{\text{FLRW}}^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (1)$$

where the function $a(t)$ is known as the scale factor in ‘cosmic’ (comoving) coordinates (t, r, θ, ϕ) , and k is the FLRW topology index, $k = (-1, 0, +1)$. The FLRW metric (1) admits the six-dimensional isometry group G that is either $SO(1,3)$, $E(3)$ or $SO(4)$, acting on the orbits $G/SO(3)$, with the spatial three-dimensional sections H^3 , E^3 or S^3 , respectively. The Weyl tensor of any FLRW metric vanishes,

$$C_{\mu\nu\lambda\rho}^{\text{FLRW}} = 0 \quad (2)$$

where $\mu, \nu, \lambda, \rho = 0, 1, 2, 3$. The early universe inflation (acceleration) means

$$\ddot{a}(t) > 0, \text{ or equivalently, } \frac{d}{dt} \left(\frac{H^{-1}}{a} \right) < 0 \quad (3)$$

where $H = \dot{a}/a$ is called Hubble function. We take $k = 0$ for simplicity. The amount of inflation (called the *e-foldings* number) is given by

$$N_e = \ln \frac{a(t_{\text{end}})}{a(t_{\text{start}})} = \int_{t_{\text{start}}}^{t_{\text{end}}} H dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi \quad (4)$$

Next, a few words about our strategy. It is well recognized now that one has to go beyond the Einstein-Hilbert action for gravity, both from the experimental viewpoint (eg., because of Dark Energy) and from the theoretical viewpoint (eg., because of the UV incompleteness of quantized Einstein gravity, and the need of its unification with the Standard Model of Elementary Particles).

In our approach, the origin of inflation is purely *geometrical*, ie. is closely related to space-time and gravity. It can be technically accomplished by taking into account the higher-order curvature terms on the left-hand-side of Einstein equations, and extending gravity to supergravity. The higher-order curvature terms are supposed to appear in the gravitational effective action of Quantum Gravity. Their derivation from Superstring Theory may be possible too. The true problem is a *selection* of those high-order curvature terms that are physically relevant or derived from a fundamental theory of Quantum Gravity.

There are many phenomenological models of inflation in the literature, which usually employ some new fields and new interactions. It is, therefore, quite reasonable and meaningful to search for the *minimal* inflationary model building, by getting most economical and viable inflationary scenario. I am going to use the one proposed the long time ago by Starobinsky (20; 21), which does not use new fields (beyond a spacetime metric) and exploits only gravitational interactions. I also assume that the general coordinate invariance in spacetime is fundamental, and it should not be sacrificed. Moreover, it should be extended to the more fundamental, local supersymmetry that is known to imply the general coordinate invariance.

On the theoretical side, the available inflationary models may be also evaluated with respect to their “cost”, ie. against what one gets from a given model in relation to what one puts in! Our approach does *not* introduce new fields, beyond those already present in gravity and supergravity. We also exploit (super)gravity interactions *only*, ie. do *not* introduce new interactions, in order to describe inflation.

Before going into details, let me address two common prejudices and objections.

The higher-order curvature terms are usually expected to be relevant near the spacetime curvature singularities. It is also quite possible that some higher-derivative gravity, subject to suitable constraints, could be the effective action to a quantized theory of gravity,¹ like eg., in String Theory. However, there are also some common doubts against the higher-derivative terms, in principle.

First, it is often argued that all higher-derivative field theories, including the higher-derivative gravity theories, have ghosts (i.e. are unphysical), because of Ostrogradski theorem (1850) in Classical Mechanics. As a matter of fact, though the presence of ghosts is a generic feature of the higher-derivative theories indeed, it is not always the case, while many explicit examples are known (Lovelock gravity, Euler densities, some *f(R)* gravity theories, etc.) — see eg., ref. (22) for more details. In our approach, the absence of ghosts and tachyons is required, and is considered as one of the main physical selection criteria for the good higher-derivative field theories.

Another common objection against the higher-derivative gravity theories is due to the fact that all the higher-order curvature terms in the action are to be suppressed by the inverse powers of M_{Pl} on dimensional reasons and, therefore, they seem to be ‘very small and negligible’. Though it is generically true, it does not mean that all the higher-order curvature terms are irrelevant at all scales much less than M_{Pl} . For instance, it appears that the *quadratic* curvature terms have *dimensionless* couplings, while they can be instrumental for an early universe inflation. A non-trivial function of R in the effective gravitational action may also ‘explain’ the Dark Energy phenomenon in the present Universe.

Cosmological inflation in supergravity is a window to High-Energy Physics beyond the Standard Model of Elementary Particles. The Starobinsky inflationary model is introduced in Sec. 2. Its classical equivalence to a scalar-tensor gravity is shown in Sec. 3, and its observational predictions for the CMB are given in Sec. 4. We review a construction of the new *F(R)* supergravity theories in Secs. 5 and 6. The *F(R)* supergravity theories are the $N = 1$ locally supersymmetric extensions of the well studied *f(R)* gravity theories in four space-time dimensions, which are often used for ‘explaining’ inflation and Dark Energy. A manifestly supersymmetric description of the *F(R)* supergravities exist in terms of $N = 1$ superfields, by using the (old) minimal Poincaré supergravity in curved superspace. We prove that any *F(R)* supergravity is classically equivalent to the particular Poincaré-type matter-coupled $N = 1$ supergravity via the superfield Legendre-Weyl-Kähler transformation. The (nontrivial) Kähler potential and the scalar superpotential of inflaton superfield are determined in terms of the original holomorphic *F(R)* function. The conditions for stability, the absence of ghosts and tachyons are also found. No-scale *F(R)* supergravity is constructed too (Sec. 7). Three different examples of the *F(R)* supergravity theories are studied in detail. The first example is devoted to recovery of the standard (pure) $N = 1$ supergravity with a negative cosmological constant from *F(R)* supergravity (Sec. 8). As the second example, a generic \mathcal{R}^2 supergravity is investigated, the existence of the AdS bound on the scalar curvature and a possibility of positive cosmological constant are discovered (Sec. 9). As

¹ To the best of my knowledge, this proposal was first formulated by A.D. Sakharov in 1967.

the third example, a simple and viable realization of chaotic inflation in supergravity is given, via an embedding of the Starobinsky inflationary model into the $F(\mathcal{R})$ supergravity (Sec. 10). Our approach does not introduce new exotic fields or new interactions, beyond those already present in (super)gravity. In Sec. 11 the nonminimal scalar-curvature couplings in gravity and supergravity, and their correspondence to $f(R)$ gravity and $F(\mathcal{R})$ supergravity, respectively, are analyzed within slow-roll inflation. Reheating and particle production are briefly discussed in Sec. 12. Our short conclusion is Sec. 13. In our outlook (Sec. 14), we emphasize the possible use of $F(\mathcal{R})$ supergravity towards solving the outstanding problems of CP -violation, the origin of baryonic asymmetry, lepto- and baryo-genesis.

2. Starobinsky minimal model of inflation

It can be argued that it is the *scalar* curvature-dependent part of the gravitational effective action that is most relevant to the large-scale dynamics $H(t)$. Here are some simple arguments. In 4 dimensions all the independent *quadratic* curvature invariants are $R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$, $R^{\mu\nu}R_{\mu\nu}$ and R^2 . However,

$$\int d^4x \sqrt{-g} \left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \right) \quad (5)$$

is topological (ie. a total derivative) for any metric, while

$$\int d^4x \sqrt{-g} \left(3R^{\mu\nu}R_{\mu\nu} - R^2 \right) \quad (6)$$

is also topological for any FLRW metric, because of eq. (2). Hence, the FLRW-relevant quadratically-generated gravity action is ($8\pi G_N = 1$)

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left(R - R^2/M^2 \right) \quad (7)$$

This action is known as the Starobinsky model (20; 21). Its equations of motion allow a stable inflationary solution, and it is an attractor! In particular, for $H \gg M$, one finds

$$H \approx \left(\frac{M}{6} \right)^2 (t_{\text{end}} - t) \quad (8)$$

It is the particular realization of chaotic inflation (ie. with chaotic initial conditions) (23), and with a Graceful Exit.

In the case of a generic gravitational action with the higher-order curvature terms, the Weyl dependence can be excluded due to eq. (2) again. A dependence upon the Ricci tensor can be also excluded since, otherwise, it would lead to the extra propagating massless spin-2 degree of freedom (in addition to a metric) described by the field $\partial\mathcal{L}/\partial R_{\mu\nu}$. The higher derivatives of the scalar curvature in the gravitational Lagrangian \mathcal{L} just lead to more propagating scalars (24), so I simply ignore them for simplicity in what follows.

3. $f(R)$ Gravity and scalar-tensor gravity

The Starobinsky model (7) is the special case of the $f(R)$ gravity theories (25; 26) having the action

$$S_f = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \tilde{f}(R) \quad (9)$$

In the absence of extra matter, the gravitational (trace) equation of motion is of the fourth order with respect to the time derivative,

$$\frac{3}{a^3} \frac{d}{dt} \left(a^3 \frac{d\tilde{f}'(R)}{dt} \right) + R\tilde{f}'(R) - 2\tilde{f}(R) = 0 \tag{10}$$

where we have used $H = \frac{\dot{a}}{a}$ and $R = -6(\dot{H} + 2H^2)$. The primes denote the derivatives with respect to R , and the dots denote the derivative with respect to t . Static de-Sitter solutions correspond to the roots of the equation $R\tilde{f}'(R) = 2\tilde{f}(R)$ (27).

The 00-component of the gravitational equations is of the third order with respect to the time derivative,

$$3H \frac{d\tilde{f}'(R)}{dt} - 3(\dot{H} + H^2)\tilde{f}'(R) - \frac{1}{2}\tilde{f}(R) = 0 \tag{11}$$

The (classical and quantum) *stability* conditions in $f(R)$ gravity are well known (25; 26), and are given by (in our notation)

$$\tilde{f}'(R) > 0 \quad \text{and} \quad \tilde{f}''(R) < 0 \tag{12}$$

respectively. The first condition (12) is needed to get a physical (non-ghost) graviton, while the second condition (12) is needed to get a physical (non-tachyonic) scalaron (see Sec. 9 for more).

Any $f(R)$ gravity is known to be classically *equivalent* to the certain scalar-tensor gravity having an (extra) propagating scalar field (28–30). The formal equivalence can be established via a Legendre-Weyl transform.

First, the $f(R)$ -gravity action (9) can be rewritten to the form

$$S_A = \frac{-1}{2\kappa^2} \int d^4x \sqrt{-g} \{AR - Z(A)\} \tag{13}$$

where the real scalar (or Lagrange multiplier) $A(x)$ is related to the scalar curvature R by the Legendre transformation:

$$R = Z'(A) \quad \text{and} \quad \tilde{f}(R) = RA(R) - Z(A(R)) \tag{14}$$

with $\kappa^2 = 8\pi G_N = M_{\text{Pl}}^{-2}$.

Next, a Weyl transformation of the metric,

$$g_{\mu\nu}(x) \rightarrow \exp \left[\frac{2\kappa\phi(x)}{\sqrt{6}} \right] g_{\mu\nu}(x) \tag{15}$$

with arbitrary field parameter $\phi(x)$ yields

$$\sqrt{-g} R \rightarrow \sqrt{-g} \exp \left[\frac{2\kappa\phi(x)}{\sqrt{6}} \right] \left\{ R - \sqrt{\frac{6}{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) \kappa - \kappa^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} \tag{16}$$

Therefore, when choosing

$$A(\kappa\phi) = \exp \left[\frac{-2\kappa\phi(x)}{\sqrt{6}} \right] \tag{17}$$

and ignoring a total derivative in the Lagrangian, we can rewrite the action to the form

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left\{ \frac{-R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2\kappa^2} \exp \left[\frac{4\kappa\phi(x)}{\sqrt{6}} \right] Z(A(\kappa\phi)) \right\} \quad (18)$$

in terms of the physical (and canonically normalized) scalar field $\phi(x)$, without any higher derivatives and ghosts. As a result, one arrives at the standard action of the real dynamical scalar field $\phi(x)$ *minimally* coupled to Einstein gravity and having the scalar potential

$$V(\phi) = -\frac{M_{\text{Pl}}^2}{2} \exp \left\{ \frac{4\phi}{M_{\text{Pl}}\sqrt{6}} \right\} Z \left(\exp \left[\frac{-2\phi}{M_{\text{Pl}}\sqrt{6}} \right] \right) \quad (19)$$

In the context of the inflationary theory, the *scalaron* (= scalar part of spacetime metric) ϕ can be identified with inflaton. This inflaton has clear origin, and may also be understood as the conformal mode of the metric over Minkowski or (A)dS vacuum.

In the Starobinsky case of $\tilde{f}(R) = R - R^2/M^2$, the inflaton scalar potential reads

$$V(y) = V_0 (e^{-y} - 1)^2 \quad (20)$$

where we have introduced the notation

$$y = \sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{Pl}}} \quad \text{and} \quad V_0 = \frac{1}{8} M_{\text{Pl}}^2 M^2 \quad (21)$$

It is worth noticing here the appearance of the inflaton vacuum energy V_0 driving inflation. The end of inflation (Graceful Exit) is also clear: the scalar potential (20) has a very flat (slow-roll) 'plateau', ending with a 'waterfall' towards the minimum (Fig. 1).

It is worth emphasizing that the inflaton (scalaron) scalar potential (20) is derived here by merely assuming the existence of the R^2 term in the gravitational action. The Newton (weak gravity) limit is not applicable to an early universe (including its inflationary stage), so that the dimensionless coefficient in front of the R^2 term does not have to be very small. It distinguishes the primordial 'dark energy' driving inflation in the early Universe from the 'Dark Energy' responsible for the present Universe acceleration.

4. Inflationary theory and observations

The *slow-roll* inflation parameters are defined by

$$\varepsilon(\phi) = \frac{1}{2} M_{\text{Pl}}^2 \left(\frac{V'}{V} \right)^2 \quad \text{and} \quad \eta(\phi) = M_{\text{Pl}}^2 \frac{V''}{V} \quad (22)$$

A necessary condition for the slow-roll approximation is the smallness of the inflation parameters

$$\varepsilon(\phi) \ll 1 \quad \text{and} \quad |\eta(\phi)| \ll 1 \quad (23)$$

The first condition implies $\ddot{a}(t) > 0$. The second one guarantees that inflation lasts long enough, via domination of the *friction* term in the inflaton equation of motion, $3H\dot{\phi} = -V'$.

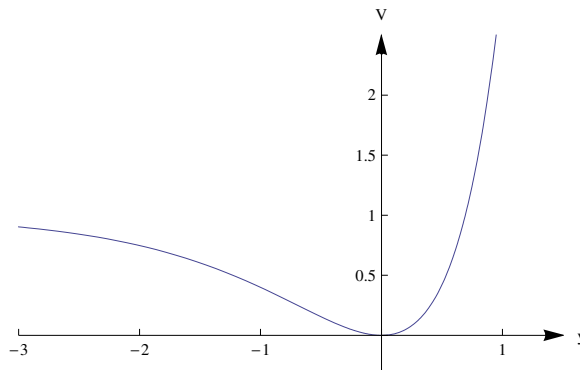


Fig. 1. The inflaton scalar potential $v(x) = (e^y - 1)^2$ in the Starobinsky model, after $y \rightarrow -y$

As is well known (13), scalar and tensor perturbations of the metric *decouple*. The scalar perturbations couple to the density of matter and radiation, so they are responsible for the inhomogeneities and anisotropies in the universe. The tensor perturbations (or gravity waves) also contribute to the CMB, while their experimental detection would tell us much more about inflation. The CMB radiation is expected to be *polarized* due to Compton scattering at the time of decoupling (31; 32).

The primordial spectrum is proportional to k^{n-1} , in terms of the comoving wave number k and the spectral index n . In theory, the slope n_s of the *scalar* power spectrum, associated with the density perturbations, $\left(\frac{\delta\rho}{\rho}\right)^2 \propto k^{n_s-1}$, is given by $n_s = 1 + 2\eta - 6\epsilon$, the slope of the *tensor* primordial spectrum, associated with gravitational waves, is $n_t = -2\epsilon$, and the *tensor-to-scalar ratio* is $r = 16\epsilon$ (see eg., ref. (13)).

It is straightforward to calculate those indices in any inflationary model with a given inflaton scalar potential. In the case of the Starobinsky model and its scalar potential (20), one finds (6; 33; 34)

$$n_s = 1 - \frac{2}{N_e} + \frac{3 \ln N_e}{2N_e^2} - \frac{2}{N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right) \tag{24}$$

and

$$r \approx \frac{12}{N_e^2} \approx 0.004 \tag{25}$$

with $N_e \approx 55$. The very small value of r is the sharp prediction of the Starobinsky inflationary model towards r -measurements in a future.

Those theoretical values are to be compared to the observed values of the CMB radiation due to the WMAP satellite mission. For instance, the most recent WMAP7 observations (14) yield

$$n_s = 0.963 \pm 0.012 \quad \text{and} \quad r < 0.24 \tag{26}$$

with the 95 % level of confidence.

The amplitude of the initial perturbations, $\Delta_R^2 = M_{\text{Pl}}^4 V / (24\pi^2 \epsilon)$, is also a physical observable, whose experimental value is known due to another Cosmic Background Explorer (COBE) satellite mission (35):

$$\left(\frac{V}{\epsilon}\right)^{1/4} = 0.027 M_{\text{Pl}} = 6.6 \times 10^{16} \text{ GeV} \tag{27}$$

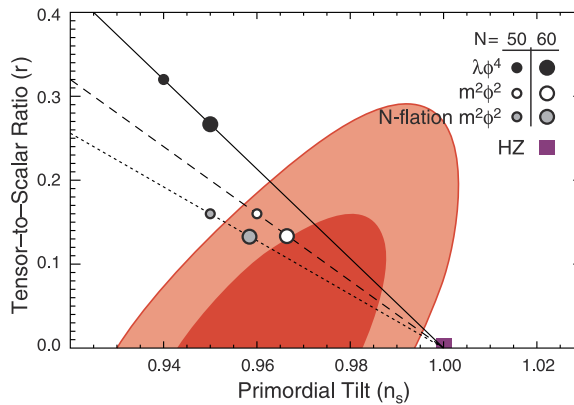


Fig. 2. Starobinsky inflation vs. $m^2\phi^2/2$ and $\lambda\phi^4$

It determines the normalization of the R^2 -term in the action (7)

$$\frac{M}{M_{\text{Pl}}} = 4 \cdot \sqrt{\frac{2}{3}} \cdot (2.7)^2 \cdot \frac{e^{-y}}{(1 - e^{-y})^2} \cdot 10^{-4} \approx (3.5 \pm 1.2) \cdot 10^{-6} \tag{28}$$

In particular, the inflaton mass is given by $M_{\text{inf}} = M/\sqrt{6}$.

The main theoretical lessons, that we can draw from the discussion above towards our next goals, are:

- (i) the main *discriminants* amongst all inflationary models are given by the values of n_s and r ;
- (ii) the Starobinsky model (1980) of chaotic inflation is very simple and economic. It uses gravity interactions only. It predicts the origin of inflaton and its scalar potential. It is still *viable* and *consistent* with all known observations. Inflaton is not charged (singlet) under the SM gauge group. The Starobinsky inflation has an end (Graceful Exit), and gives the simple explanation to the WMAP-observed value of n_s . The *key* difference of Starobinsky inflation from the other standard inflationary models (having $\frac{1}{2}m^2\phi^2$ or $\lambda\phi^4$ scalar potentials) is the very low value of r — see the standard Fig. 2 for a comparison and ref. (36) for details. A discovery of primordial gravitational waves and precision measurements of the value of r (if $r \geq 0.1$) with the accuracy of 0.5% may happen due to the ongoing PLANCK satellite mission (37);
- (iii) the viable inflationary models, based on $\tilde{f}(R) = R + \hat{f}(R)$ gravity, turn out to be close to the simplest Starobinsky model (over the range of R relevant to inflation), with $\hat{f}(R) \approx R^2 A(R)$ and the slowly varying function $A(R)$ in the sense

$$|A'(R)| \ll \frac{A(R)}{R} \quad \text{and} \quad |A''(R)| \ll \frac{A(R)}{R^2} \tag{29}$$

5. Supergravity and superspace

Supersymmetry (SUSY) is the symmetry between bosons and fermions. SUSY is the natural extension of Poincaré symmetry, and is well motivated in HEP beyond the SM. Supersymmetry is also needed for consistency of strings. Supergravity (SUGRA) is the theory of *local* supersymmetry that implies general coordinate invariance. In other words,

considering inflation with supersymmetry necessarily leads to supergravity. As a matter of fact, most of studies of superstring- and brane-cosmology are also based on their effective description in the 4-dimensional $N = 1$ supergravity.

It is not our purpose here to give a detailed account of SUSY and SUGRA, because of the existence of several textbooks — see e.g., refs. (38–40). In this Section I recall only the basic facts about $N = 1$ supergravity in four spacetime dimensions, which are needed here.

A concise and manifestly supersymmetric description of SUGRA is given by Superspace. In this section the natural units $c = \hbar = \kappa = 1$ are used.

Supergravity needs a curved superspace. However, they are not the same, because one has to reduce the field content to the minimal one corresponding to off-shell supergravity multiplets. It is done by imposing certain constraints on the supertorsion tensor in curved superspace (38–40). An off-shell supergravity multiplet has some extra (auxiliary) fields with noncanonical dimensions, in addition to physical spin-2 field (metric) and spin-3/2 field (gravitino). It is worth mentioning that imposing the off-shell constraints is independent upon writing a supergravity action.

One may work either in a full superspace or in a chiral one. There are certain advantages of using the chiral superspace, because it helps us to keep the auxiliary fields unphysical (i.e. nonpropagating).

The chiral superspace density (in the supersymmetric gauge-fixed form) reads

$$\mathcal{E}(x, \theta) = e(x) \left[1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x) \right], \tag{30}$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi_\alpha^a = e_\mu^a \psi_\alpha^\mu$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle greek letters $\mu, \nu, \dots = 0, 1, 2, 3$ for curved spacetime vector indices, the lower case early latin letters $a, b, \dots = 0, 1, 2, 3$ for flat (target) space vector indices, and the lower case early greek letters $\alpha, \beta, \dots = 1, 2$ for chiral spinor indices.

A solution to the superspace Bianchi identities together with the constraints defining the $N = 1$ Poincaré-type *minimal* supergravity theory results in only *three* covariant tensor superfields \mathcal{R} , \mathcal{G}_a and $\mathcal{W}_{\alpha\beta\gamma}$, subject to the off-shell relations (38–40):

$$\mathcal{G}_a = \bar{\mathcal{G}}_a, \quad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}, \quad \bar{\nabla}_{\dot{\alpha}} \mathcal{R} = \bar{\nabla}_{\dot{\alpha}} \mathcal{W}_{\alpha\beta\gamma} = 0, \tag{31}$$

and

$$\bar{\nabla}_{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}} = \nabla_{\alpha} \mathcal{R}, \quad \nabla^{\gamma} \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \nabla_{\alpha} \mathcal{G}_{\beta\dot{\alpha}} + \frac{i}{2} \nabla_{\beta} \mathcal{G}_{\alpha\dot{\alpha}}, \tag{32}$$

where $(\nabla_{\alpha}, \bar{\nabla}_{\dot{\alpha}})$ stand for the curved superspace $N = 1$ supercovariant derivatives, and the bars denote complex conjugation.

The covariantly chiral complex scalar superfield \mathcal{R} has the scalar curvature R as the coefficient at its θ^2 term, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor, $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2}g_{\mu\nu}R$, as the coefficient at its $\theta\sigma^a\bar{\theta}$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the self-dual part of the Weyl tensor $C_{\alpha\beta\gamma\delta}$ as the coefficient at its linear θ^{δ} -dependent term.

A generic Lagrangian representing the supergravitational effective action in (full) superspace, reads

$$\mathcal{L} = \mathcal{L}(\mathcal{R}, \mathcal{G}, \mathcal{W}, \dots) \tag{33}$$

where the dots stand for arbitrary supercovariant derivatives of the superfields.

The Lagrangian (33) in its most general form is, however, unsuitable for physical applications, not only because it is too complicated, but just because it generically leads to propagating auxiliary fields, which break the balance of the bosonic and fermionic degrees of freedom. The important physical condition of keeping the supergravity auxiliary fields to be truly auxiliary (ie. nonphysical or nonpropagating) in field theories with the higher derivatives was dubbed the ‘auxiliary freedom’ in refs. (41; 42). To get the supergravity actions with the ‘auxiliary freedom’, we will use a chiral (curved) superspace.

6. $F(\mathcal{R})$ supergravity in superspace

Let us first concentrate on the scalar-curvature-sector of a generic higher-derivative supergravity (33), which is most relevant to the FRLW cosmology, by ignoring the tensor curvature superfields $\mathcal{W}_{\alpha\beta\gamma}$ and $\mathcal{G}_{\alpha\dot{\alpha}}$, as well as the derivatives of the scalar superfield \mathcal{R} , like that in Sec. 2. Then we arrive at the chiral superspace action

$$S_F = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \quad (34)$$

governed by a chiral or *holomorphic* function $F(\mathcal{R})$.² Besides having the manifest local $N = 1$ supersymmetry, the action (34) has the auxiliary freedom since the auxiliary field B does not propagate. It distinguishes the action (34) from other possible truncations of eq. (33). The action (34) gives rise to the spacetime *torsion* generated by gravitino, while its bosonic terms have the form

$$S_f = -\frac{1}{2} \int d^4x \sqrt{-g} \tilde{f}(R) \quad (35)$$

Hence, eq. (34) can also be considered as the locally $N = 1$ supersymmetric extension of the $f(R)$ -type gravity (Sec. 3). However, in the context of supergravity, the choice of possible bosonic functions $\tilde{f}(R)$ is very restrictive (see Secs. 9 and 10).

The superfield action (34) is classically equivalent to

$$S_V = \int d^4x d^2\theta \mathcal{E} [\mathcal{Z}\mathcal{R} - V(\mathcal{Z})] + \text{H.c.} \quad (36)$$

with the covariantly chiral superfield \mathcal{Z} as the Lagrange multiplier superfield. Varying the action (36) with respect to \mathcal{Z} gives back the original action (34) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})) \quad (37)$$

where the function $\mathcal{Z}(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}) \quad (38)$$

Equations (37) and (38) define the superfield Legendre transform, and imply

$$F'(\mathcal{R}) = \mathcal{Z}(\mathcal{R}) \quad \text{and} \quad F''(\mathcal{R}) = \mathcal{Z}'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))} \quad (39)$$

where $V'' = d^2V/d\mathcal{Z}^2$. The second formula (39) is the *duality* relation between the supergravitational function F and the chiral superpotential V .

² The similar component field construction, by the use of the 4D, $N = 1$ superconformal tensor calculus, was given in ref. (43).

A supersymmetric (local) Weyl transform of the action (36) can be done entirely in superspace. In terms of the field components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal) *S*-supersymmetry transformation (44). The chiral density superfield \mathcal{E} appears to be the chiral *compensator* of the super-Weyl transformations,

$$\mathcal{E} \rightarrow e^{3\Phi} \mathcal{E} \tag{40}$$

whose parameter Φ is an arbitrary covariantly chiral superfield, $\bar{\nabla}_{\dot{\alpha}} \Phi = 0$. Under the transformation (40) the covariantly chiral superfield \mathcal{R} transforms as

$$\mathcal{R} \rightarrow e^{-2\Phi} \left(\mathcal{R} - \frac{1}{4} \nabla^2 \right) e^{\Phi} \tag{41}$$

The super-Weyl chiral superfield parameter Φ can be traded for the chiral Lagrange multiplier \mathcal{Z} by using a generic gauge condition

$$\mathcal{Z} = \mathcal{Z}(\Phi) \tag{42}$$

where $\mathcal{Z}(\Phi)$ is a holomorphic function of Φ . It results in the action

$$S_{\Phi} = \int d^4x d^4\theta E^{-1} e^{\Phi + \bar{\Phi}} [\mathcal{Z}(\Phi) + \text{H.c.}] - \int d^4x d^2\theta \mathcal{E} e^{3\Phi} V(\mathcal{Z}(\Phi)) + \text{H.c.} \tag{43}$$

Equation (43) has the *standard* form of the action of a chiral matter superfield coupled to supergravity,

$$S[\Phi, \bar{\Phi}] = \int d^4x d^4\theta E^{-1} \Omega(\Phi, \bar{\Phi}) + \left[\int d^4x d^2\theta \mathcal{E} P(\Phi) + \text{H.c.} \right] \tag{44}$$

in terms of the non-chiral potential $\Omega(\Phi, \bar{\Phi})$ and the chiral superpotential $P(\Phi)$. In our case (43) we find

$$\Omega(\Phi, \bar{\Phi}) = e^{\Phi + \bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] , \quad P(\Phi) = -e^{3\Phi} V(\mathcal{Z}(\Phi)) \tag{45}$$

The *Kähler* potential $K(\Phi, \bar{\Phi})$ is given by

$$K = -3 \ln\left(-\frac{\Omega}{3}\right) \quad \text{or} \quad \Omega = -3e^{-K/3} \tag{46}$$

so that the action (44) is invariant under the supersymmetric (local) Kähler-Weyl transformations

$$K(\Phi, \bar{\Phi}) \rightarrow K(\Phi, \bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) , \quad P(\Phi) \rightarrow -e^{-\Lambda(\Phi)} P(\Phi) \tag{47}$$

with the chiral superfield parameter $\Lambda(\Phi)$. It follows that

$$\mathcal{E} \rightarrow e^{\Lambda(\Phi)} \mathcal{E} \tag{48}$$

The scalar potential in terms of the usual fields is given by the standard formula (45)

$$\mathcal{V}(\phi, \bar{\phi}) = e^K \left\{ \left| \frac{\partial P}{\partial \Phi} + \frac{\partial K}{\partial \Phi} P \right|^2 - 3 |P|^2 \right\} \tag{49}$$

where all the superfields are restricted to their leading field components, $\Phi| = \phi(x)$, and we have introduced the notation

$$\left| \frac{\partial P}{\partial \Phi} + \frac{\partial K}{\partial \Phi} P \right|^2 \equiv |D_\Phi P|^2 = D_\Phi P (K_{\Phi\bar{\Phi}}^{-1}) \bar{D}_\Phi \bar{P} \quad (50)$$

with $K_{\Phi\bar{\Phi}} = \partial^2 K / \partial \Phi \partial \bar{\Phi}$. Equation (49) can be simplified by making use of the Kähler-Weyl invariance (47) that allows one to choose a gauge

$$P = 1 \quad (51)$$

It is equivalent to the well known fact that the scalar potential (49) is actually governed by the *single* (Kähler-Weyl-invariant) potential

$$G(\Phi, \bar{\Phi}) = \Omega + \ln |P|^2 \quad (52)$$

In our case (45) we find

$$G = e^{\Phi+\bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] + 3(\Phi + \bar{\Phi}) + \ln(V(\mathcal{Z}(\Phi))) + \ln(\bar{V}(\bar{\mathcal{Z}}(\bar{\Phi}))) \quad (53)$$

So let us choose a gauge by the condition

$$3\Phi + \ln(V(\mathcal{Z}(\Phi))) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = e^{-3\Phi} \quad (54)$$

that is equivalent to eq. (51). Then the G-potential (53) gets simplified to

$$G = e^{\Phi+\bar{\Phi}} [\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})] \quad (55)$$

There is the correspondence between a holomorphic function $F(\mathcal{R})$ in the supergravity action (34) and a holomorphic function $\mathcal{Z}(\Phi)$ defining the scalar potential (49),

$$\mathcal{V} = e^G \left[\left(\frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} \right)^{-1} \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}} - 3 \right] \quad (56)$$

in the classically equivalent scalar-tensor supergravity.

To the end of this section, I would like to comment on the standard way of the inflationary model building by a choice of $K(\Phi, \bar{\Phi})$ and $P(\Phi)$ — see eg., ref. (46) for a recent review.

The factor $\exp(K/M_{\text{Pl}}^2)$ in the F -type scalar potential (49) of the chiral matter-coupled supergravity, in the case of the *canonical* Kähler potential, $K \propto \bar{\Phi}\Phi$, results in the scalar potential $V \propto \exp(|\Phi|^2/M_{\text{Pl}}^2)$ that is too steep to support chaotic inflation. Actually, it also implies $\eta \approx 1$ or, equivalently, $M_{\text{inflaton}}^2 \approx V_0/M_{\text{Pl}}^2 \approx H^2$. It is known as the η -*problem* in supergravity (47).

As is clear from our discussion above, the η -problem is not really a supergravity problem, but it is the problem associated with the choice of the canonical Kähler potential for an inflaton superfield. The Kähler potential in supergravity is a (Kähler) gauge-dependent quantity, and its quantum renormalization is not under control. Unlike the one-field inflationary models, a generic Kähler potential is a function of at least two fields, so it implies a nonvanishing curvature in the target space of the *non-linear sigma-model* associated with the Kähler kinetic

term.³ Hence, a generic Kähler potential cannot be brought to the canonical form by a field redefinition.

To solve the η -problem associated with the simplest (naive) choice of the Kähler potential, one may assume that the Kähler potential K possesses some shift symmetries (leading to its *flat* directions), and then choose inflaton in one such flat direction (49). However, in order to get inflation that way, one also has to add (“by hand”) the proper inflaton superpotential breaking the initially introduced shift symmetry, and then stabilize the inflationary trajectory with the help of yet another matter superfield.

The possible alternative is the D -term mechanism (50), where inflation is generated in the matter *gauge* sector and, as a result, is highly sensitive to the gauge charges.

It is worth mentioning that in the (perturbative) superstring cosmology one gets the Kähler potential (see e.g., refs. (51; 52))

$$K \propto \log(\text{moduli polynomial})_{CY} \tag{57}$$

over a Calabi-Yau (CY) space in the type-IIB superstring compactification, thus avoiding the η -problem but leading to a plenty of choices (embarrassment of riches!) in the String Landscape.

Finally, one still has to accomplish stability of a given inflationary model in supergravity against quantum corrections. Such corrections can easily spoil the flatness of the inflaton potential. The Kähler kinetic term is not protected against quantum corrections, because it is given by a full superspace integral (unlike the chiral superpotential term). The $F(\mathcal{R})$ supergravity action (34) is given by a *chiral* superspace integral, so that it is protected against the quantum corrections given by full superspace integrals.

To conclude this section, we claim that an $N = 1$ locally supersymmetric extension of $f(R)$ gravity is possible. It is non-trivial because the auxiliary freedom has to be preserved. The new supergravity action (34) is classically equivalent to the standard $N = 1$ Poincaré supergravity coupled to a *dynamical* chiral matter superfield, whose Kähler potential and the superpotential are dictated by a single *holomorphic* function. Inflaton can be identified with the real scalar field component of that chiral matter superfield originating from the supervielbein.

It is worth noticing that the action (34) allows a natural extension in chiral curved superspace, due to the last equation (31), namely,

$$S_{\text{ext}} = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}, \mathcal{W}^2) + \text{H.c.} \tag{58}$$

where $\mathcal{W}_{\alpha\beta\gamma}$ is the $N = 1$ covariantly-chiral Weyl superfield of the $N = 1$ superspace supergravity, and $\mathcal{W}^2 = \mathcal{W}_{\alpha\beta\gamma} \mathcal{W}^{\alpha\beta\gamma}$. The action (58) also has the auxiliary freedom. In Superstring Theory, the Weyl-tensor-dependence of the gravitational effective action is unambiguously determined by the superstring scattering amplitudes or by the super-Weyl invariance of the corresponding non-linear sigma-model (see eg., ref. (48)).

A possible connection of $F(\mathcal{R})$ supergravity to the *Loop Quantum Gravity* was investigated in ref. (3).

³ See eg., ref. (48) for more about the non-linear sigma-models.

7. No-scale $F(\mathcal{R})$ supergravity

In this section we would like to investigate a possibility of spontaneous supersymmetry breaking, without fine tuning, by imposing the condition of the vanishing scalar potential. Those no-scale supergravities are the starting point of many phenomenological applications of supergravity in HEP and inflationary theory, including string theory applications — see eg., refs. (53; 54) and references therein.

The no-scale supergravity arises by demanding the scalar potential (49) to vanish. It results in the vanishing cosmological constant without fine-tuning (55). The no-scale supergravity potential G has to obey the non-linear 2nd-order partial differential equation, which follows from eq. (56),

$$3 \frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} = \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}} \quad (59)$$

A gravitino mass $m_{3/2}$ is given by the vacuum expectation value (39)

$$m_{3/2} = \left\langle e^{G/2} \right\rangle \quad (60)$$

so that the spontaneous supersymmetry breaking scale can be chosen at will.

The well known exact solution to eq. (59) is given by

$$G = -3 \log(\Phi + \bar{\Phi}) \quad (61)$$

In the recent literature, the no-scale solution (61) is usually modified by other terms, in order to achieve the universe with a positive cosmological constant — see e.g., the KKL mechanism (56).

To appreciate the difference between the standard no-scale supergravity solution and our 'modified' supergravity, it is worth noticing that the Ansatz (61) is not favoured by our potential (55). In our case, demanding eq. (59) gives rise to the 1st-order non-linear partial differential equation

$$3 \left(e^{\bar{\Phi}} X' + e^{\Phi} \bar{X}' \right) = \left| e^{\bar{\Phi}} X' + e^{\Phi} \bar{X}' \right|^2 \quad (62)$$

where we have introduced the notation

$$\mathcal{Z}(\Phi) = e^{-\Phi} X(\Phi), \quad X' = \frac{dX}{d\Phi} \quad (63)$$

in order to get the differential equation in its most symmetric and concise form.

Accordingly, the gravitino mass (60) is given by

$$m_{3/2} = \left\langle \exp \frac{1}{2} \left(e^{\Phi} X + e^{\bar{\Phi}} \bar{X} \right) \right\rangle \quad (64)$$

I am not aware of any non-trivial holomorphic exact solution to eq. (62). However, should it obey a holomorphic differential equation of the form

$$X' = e^{\Phi} g(X, \Phi) \quad (65)$$

with a holomorphic function $g(X, \Phi)$, eq. (62) gives rise to the functional equation

$$3(g + \bar{g}) = \left| e^{\Phi} g + \bar{X}' \right|^2 \quad (66)$$

Being restricted to the real variables $\Phi = \bar{\Phi} \equiv y$ and $X = \bar{X} \equiv x$, eq. (62) reads

$$6x' = e^y(x' + x)^2, \quad \text{where} \quad x' = \frac{dx}{dy} \tag{67}$$

This equation can be integrated after a change of variables,

$$x = e^{-y}u, \tag{68}$$

and it leads to a quadratic equation with respect to $u' = du/dy$,

$$(u')^2 - 6u' + 6u = 0 \tag{69}$$

It follows

$$y = \int^u \frac{d\zeta}{3 \pm \sqrt{3(3 - 2\zeta)}} = \mp \sqrt{1 - \frac{2}{3}u} + \ln \left(\sqrt{3(3 - 2u)} \pm 3 \right) + C. \tag{70}$$

8. Fields from superfields in *F*(\mathcal{R}) supergravity

For simplicity, we set all fermionic fields to zero, when passing to the field components. It greatly simplifies most of the field equations, but makes supersymmetry to be manifestly broken (however, SUSY is restored after adding all those fermionic terms back to the action). Applying the standard superspace chiral density formula (38–40)

$$\int d^4x d^2\theta \mathcal{E} \mathcal{L} = \int d^4x e \{ \mathcal{L}_{\text{last}} + B \mathcal{L}_{\text{first}} \} \tag{71}$$

to the action (34) yields its bosonic part in the form

$$(-g)^{-1/2} L_{\text{bos}} \equiv f(R, \bar{R}; X, \bar{X}) = F'(\bar{X}) \left[\frac{1}{3} R_* + 4 \bar{X} X \right] + 3XF(\bar{X}) + \text{H.c.} \tag{72}$$

where the primes denote differentiation with respect to a given argument. We have used the notation

$$X = \frac{1}{3}B \quad \text{and} \quad R_* = R + \frac{i}{2} \epsilon^{abcd} R_{abcd} \equiv R + i\bar{R} \tag{73}$$

The \bar{R} does not vanish in *F*(\mathcal{R}) supergravity, and it represents the *axion* field that is the pseudo-scalar superpartner of real scalaron field in our construction. Varying eq. (72) with respect to the auxiliary fields X and \bar{X} ,

$$\frac{\partial L_{\text{bos}}}{\partial X} = \frac{\partial L_{\text{bos}}}{\partial \bar{X}} = 0 \tag{74}$$

gives rise to the algebraic equations on the auxiliary fields,

$$3\bar{F} + X(4\bar{F}' + 7F') + 4\bar{X}XF'' + \frac{1}{3}F''R_* = 0 \tag{75}$$

and its conjugate

$$3F + \bar{X}(4F' + 7\bar{F}') + 4\bar{X}X\bar{F}'' + \frac{1}{3}\bar{F}''\bar{R}_* = 0 \tag{76}$$

where $F = F(X)$ and $\bar{F} = \bar{F}(\bar{X})$. The algebraic equations (75) and (76) cannot be explicitly solved for X in a generic *F*(\mathcal{R}) supergravity.

To recover the standard (pure) supergravity in our approach, let us consider the simple special case when

$$F'' = 0 \quad \text{or, equivalently,} \quad F(\mathcal{R}) = f_0 - \frac{1}{2}f_1\mathcal{R} \quad (77)$$

with some complex constants f_0 and f_1 , where $\text{Re}f_1 > 0$. Then eq. (75) is easily solved as

$$\bar{X} = \frac{3f_0}{5(\text{Re}f_1)} \quad (78)$$

Substituting this solution back into the Lagrangian (72) yields

$$L = -\frac{1}{3}(\text{Re}f_1)R + \frac{9|f_0|^2}{5(\text{Re}f_1)} \equiv -\frac{1}{2}M_{\text{Pl}}^2 R - \Lambda \quad (79)$$

where we have introduced the reduced Planck mass M_{Pl} , and the cosmological constant Λ as

$$\text{Re}f_1 = \frac{3}{2}M_{\text{Pl}}^2 \quad \text{and} \quad \Lambda = \frac{-6|f_0|^2}{5M_{\text{Pl}}^2} \quad (80)$$

It is the standard pure supergravity with a *negative* cosmological constant (38–40).

9. Generic \mathcal{R}^2 supergravity, and AdS bound

The simplest non-trivial $F(\mathcal{R})$ supergravity is obtained by choosing $F'' = \text{const.} \neq 0$ that leads to the \mathcal{R}^2 -supergravity defined by a generic *quadratic* polynomial in terms of the scalar supercurvature (8).

Let us recall that the stability conditions in $f(R)$ -gravity are given by eqs. (12) in the notation (9). In the notation (72) used here, ie. when $f(R) = -\frac{1}{2}M_{\text{Pl}}^2\tilde{f}(R)$, one gets the opposite signs,

$$f'(R) < 0 \quad (81)$$

and

$$f''(R) > 0 \quad (82)$$

The first (classical stability) condition (81) is related to the sign factor in front of the Einstein-Hilbert term (linear in R) in the $f(R)$ -gravity action, and it ensures that graviton is not a ghost. The second (quantum stability) condition (82) guarantees that scalaron is not a tachyon.

Being mainly interested in the inflaton part of the bosonic $f(R)$ -gravity action that follows from eq. (72), we set both gravitino and axion to zero, which also implies $R_* = R$ and a *real* X . In $F(R)$ supergravity the stability condition (81) is to be replaced by a stronger condition,

$$F'(X) < 0 \quad (83)$$

It is easy to verify that eq. (81) follows from eq. (83) because of eq. (74). Equation (83) also ensures the classical stability of the bosonic $f(R)$ gravity embedding into the full $F(\mathcal{R})$ supergravity against small fluctuations of the axion field.

Let's now investigate the most general non-trivial Ansatz (with $F'' = \text{const.} \neq 0$) for the $F(R)$ supergravity function in the form

$$F(\mathcal{R}) = f_0 - \frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 \quad (84)$$

with three coupling constants f_0, f_1 and f_2 . We will take all of them to be real, since we will ignore this potential source of CP -violation here (see, however, the Outlook). As regards the mass dimensions of the quantities introduced, we have

$$[F] = [f_0] = 3, \quad [R] = [f_1] = 2, \quad \text{and} \quad [\mathcal{R}] = [f_2] = 1 \tag{85}$$

The bosonic Lagrangian (72) with the function (84) reads

$$(-g)^{-1/2} L_{\text{bos}} = 11f_2X^3 - 7f_1X^2 + \left(\frac{2}{3}f_2R + 6f_0\right)X - \frac{1}{3}f_1R \tag{86}$$

Hence, the auxiliary field equation (74) takes the form of a *quadratic* equation,

$$\frac{33}{2}f_2X^2 - 7f_1X + \frac{1}{3}Rf_2 + 3f_0 = 0 \tag{87}$$

whose solution is given by

$$X_{\pm} = \frac{7}{3 \cdot 11} \left[\frac{f_1}{f_2} \pm \sqrt{\frac{2 \cdot 11}{7^2} (R_{\text{max}} - R)} \right] \tag{88}$$

where we have introduced the *maximal* scalar curvature

$$R_{\text{max}} = \frac{7^2}{2 \cdot 11} \frac{f_1^2}{f_2^2} - 3^2 \frac{f_0}{f_2} \tag{89}$$

Equation (88) obviously implies the automatic bound on the scalar curvature (from one side only). In our notation, it corresponds to the (AdS) bound on the scalar curvature from above,

$$R < R_{\text{max}} \tag{90}$$

The existence of the built-in maximal (upper) scalar curvature (or the AdS bound) is a nice bonus of our construction. It is similar to the factor $\sqrt{1 - v^2/c^2}$ in Special Relativity. Yet another close analogy comes from the *Born-Infeld* non-linear extension of Maxwell electrodynamics, whose (dual) Hamiltonian is proportional to (48)

$$\left(1 - \sqrt{1 - \vec{E}^2/E_{\text{max}}^2 - \vec{H}^2/H_{\text{max}}^2 + (\vec{E} \times \vec{H})^2/E_{\text{max}}^2 H_{\text{max}}^2} \right) \tag{91}$$

in terms of the electric and magnetic fields \vec{E} and \vec{H} , respectively, with their maximal values. For instance, in String Theory one has $E_{\text{max}} = H_{\text{max}} = (2\pi\alpha')^{-1}$ (48).

Substituting the solution (88) back into eq. (86) yields the corresponding $f(R)$ -gravity Lagrangian

$$f_{\pm}(R) = \frac{2 \cdot 7}{11} \frac{f_0 f_1}{f_2} - \frac{2 \cdot 7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2} - \frac{19}{3^2 \cdot 11} f_1 R \mp \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) (R_{\text{max}} - R)^{3/2} \tag{92}$$

Expanding eq. (92) into power series of R yields

$$f_{\pm}(R) = -\Lambda_{\pm} - a_{\pm}R + b_{\pm}R^2 + \mathcal{O}(R^3) \tag{93}$$

whose coefficients are given by

$$\Lambda_{\pm} = \frac{2 \cdot 7}{3^2 \cdot 11} f_1 \left(R_{\max} - \frac{7^2}{2 \cdot 3 \cdot 11} \frac{f_1^2}{f_2^2} \right) \pm \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) R_{\max}^{3/2} \quad (94)$$

$$a_{\pm} = \frac{19}{3^2 \cdot 11} f_1 \mp \sqrt{\frac{2}{11} R_{\max}} \left(\frac{2}{3^2} f_2 \right) \quad (95)$$

and

$$b_{\pm} = \mp \sqrt{\frac{2}{11 R_{\max}}} \left(\frac{f_2}{2 \cdot 3^2} \right) \quad (96)$$

Those equations greatly simplify when $f_0 = 0$. One finds (5; 8)

$$f_{\pm}^{(0)}(R) = \frac{-5 \cdot 17 M_{\text{Pl}}^2}{2 \cdot 3^2 \cdot 11} R + \frac{2 \cdot 7}{3^2 \cdot 11} M_{\text{Pl}}^2 (R - R_{\max}) \left[1 \pm \sqrt{1 - R/R_{\max}} \right] \quad (97)$$

where we have chosen

$$f_1 = \frac{3}{2} M_{\text{Pl}}^2 \quad (98)$$

in order to get the standard normalization of the Einstein-Hilbert term that is linear in R . Then, in the limit $R_{\max} \rightarrow +\infty$, both functions $f_{\pm}^{(0)}(R)$ reproduce General Relativity. In another limit $R \rightarrow 0$, one finds a *vanishing* or *positive* cosmological constant,

$$\Lambda_{-}^{(0)} = 0 \quad \text{and} \quad \Lambda_{+}^{(0)} = \frac{2^2 \cdot 7}{3^2 \cdot 11} M_{\text{Pl}}^2 R_{\max} \quad (99)$$

The stability conditions are given by eqs. (81), (82) and (83), while the 3rd condition implies the 2nd one. In our case (92) we have

$$f'_{\pm}(R) = -\frac{19}{3^2 \cdot 11} f_1 \pm \sqrt{\frac{2}{11}} \left(\frac{2}{3^2} f_2 \right) \sqrt{R_{\max} - R} < 0 \quad (100)$$

and

$$f''_{\pm}(R) = \mp \left(\frac{f_2}{3^2} \right) \sqrt{\frac{2}{11(R_{\max} - R)}} > 0 \quad (101)$$

while eqs. (83), (84) and (88) yield

$$\pm \sqrt{\frac{2 \cdot 11}{7^2} (R_{\max} - R)} < \frac{19}{2 \cdot 7} \frac{f_1}{f_2} \quad (102)$$

It follows from eq. (101) that

$$f_2^{(+)} < 0 \quad \text{and} \quad f_2^{(-)} > 0 \quad (103)$$

Then the stability condition (82) is obeyed for any value of R .

As regards the $(-)$ -case, there are *two* possibilities depending upon the sign of f_1 . Should f_1 be *positive*, all the remaining stability conditions are automatically satisfied, ie. in the case of both $f_2^{(-)} > 0$ and $f_1^{(-)} > 0$.

Should f_1 be *negative*, $f_1^{(-)} < 0$, we find that the remaining stability conditions (100) and (102) are *the same*, as they should, while they are both given by

$$R < R_{\max} - \frac{19^2}{2^3 \cdot 11} \frac{f_1^2}{f_2^2} = -\frac{3 \cdot 5}{2^3 \cdot 11} \frac{f_1^2}{f_2^2} - 3^2 \frac{f_0}{f_2} \equiv R_{\max}^{\text{ins}} \tag{104}$$

As regards the (+)-case, eq. (102) implies that f_1 should be *negative*, $f_1 < 0$, whereas then eqs. (100) and (102) result in *the same* condition (104) again.

Since $R_{\max}^{\text{ins}} < R_{\max}$, our results imply that the instability happens *before* R reaches R_{\max} in all cases with negative f_1 .

As regards the particularly simple case (97), the stability conditions allow us to choose the lower sign only.

A different example arises with a negative f_1 . When choosing the lower sign (ie. a positive f_2) for definiteness, we find

$$f_-(R) = -\frac{2 \cdot 7}{11} f_0 \left| \frac{f_1}{f_2} \right| + \frac{2 \cdot 7^3}{3^3 \cdot 11^2} \left| \frac{f_1^3}{f_2^3} \right| + \frac{19}{3^2 \cdot 11} |f_1| R + \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) (R_{\max} - R)^{3/2} \tag{105}$$

Demanding the standard normalization of the Einstein-Hilbert term in this case implies

$$R_{\max} = \frac{3^4 \cdot 11}{2^3 f_2^2} \left(\frac{M_{\text{Pl}}^2}{2} + \frac{19}{3^2 \cdot 11} |f_1| \right)^2 \tag{106}$$

where we have used eq. (95). It is easy to verify by using eq. (94) that the cosmological constant is always *negative* in this case, and the instability bound (104) is given by

$$R_{\max}^{\text{ins}} = \frac{3^4 \cdot 11 M_{\text{Pl}}^2}{2^3 f_2^2} \left(\frac{M_{\text{Pl}}^2}{2^2} + \frac{19 |f_1|}{3^2 \cdot 11} \right) < R_{\max} \tag{107}$$

The $f_-(R)$ function of eq. (92) can be rewritten to the form

$$f(R) = \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^3} - \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\max} - \frac{19}{3^2 \cdot 11} f_1 R + f_2 \sqrt{\frac{2^5}{3^6 \cdot 11}} (R_{\max} - R)^{3/2} \tag{108}$$

where we have used eq. (89). There are *three* physically different regimes:

(i) the *high-curvature regime*, $R < 0$ and $|R| \gg R_{\max}$. Then eq. (108) implies

$$f(R) \approx -\Lambda_h - a_h R + c_h |R|^{3/2} \tag{109}$$

whose coefficients are given by

$$\begin{aligned} \Lambda_h &= \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\max} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^3} , \\ a_h &= \frac{19}{3^2 \cdot 11} f_1 , \\ c_h &= \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) \end{aligned} \tag{110}$$

(ii) the *low-curvature regime*, $|R/R_{\max}| \ll 1$. Then eq. (108) implies

$$f(R) \approx -\Lambda_l - a_l R, \tag{111}$$

whose coefficients are given by

$$\begin{aligned} \Lambda_l &= \Lambda_h - \sqrt{\frac{2R_{\max}^3}{11}} \left(\frac{2^2}{3^3} f_2 \right), \\ a_l &= a_h + \sqrt{\frac{2R_{\max}}{11}} \left(\frac{2}{3^2} f_2 \right) = a_- = \frac{M_{\text{Pl}}^2}{2}, \end{aligned} \tag{112}$$

where we have used eq. (95).

(iii) the *near-the-bound regime* (assuming that no instability happens before it), $R = R_{\max} + \delta R$, $\delta R < 0$, and $|\delta R/R_{\max}| \ll 1$. Then eq. (108) implies

$$f(R) \approx -\Lambda_b + a_b |\delta R| + c_b |\delta R|^{3/2} \tag{113}$$

whose coefficients are

$$\begin{aligned} \Lambda_b &= \frac{1}{3} f_1 R_{\max} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2}, \\ a_b &= a_h, \\ c_b &= \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) \end{aligned} \tag{114}$$

The cosmological dynamics may be either directly derived from the gravitational equations of motion in the $f(R)$ -gravity with a given function $f(R)$, or just read off from the form of the corresponding scalar potential of a scalaron (see below). For instance, as was demonstrated in ref. (5) for the special case $f_0 = 0$, a cosmological expansion is possible in the regime (i) towards the regime (ii), and then, perhaps, to the regime (iii) unless an instability occurs.

However, one should be careful since our toy-model (84) does not pretend to be viable in the low-curvature regime, eg., for the present Universe. Nevertheless, if one wants to give it some physical meaning there, by identifying it with General Relativity, then one should also fine-tune the cosmological constant Λ_l in eq. (112) to be “small” and positive. We find that it amounts to

$$R_{\max} \approx \frac{3^4 \cdot 7^2 \cdot 11}{2^5 \cdot 19^2} \frac{M_{\text{Pl}}^4}{f_2^2} \equiv R_{\Lambda=0} \tag{115}$$

with the actual value of R_{\max} to be “slightly” above of that bound, $R_{\max} > R_{\Lambda=0}$. It is also possible to have the vanishing cosmological constant, $\Lambda_l = 0$, when choosing $R_{\max} = R_{\Lambda=0}$. It is worth mentioning that it relates the values of R_{\max} and f_2 .

The particular \mathcal{R}^2 -supergravity model (with $f_0 = 0$) was introduced in ref. (5) in an attempt to get viable embedding of the Starobinsky model into $F(\mathcal{R})$ -supergravity. However, it failed because, as was found in ref. (5), the higher-order curvature terms cannot be ignored in eq. (97), ie. the R^n -terms with $n \geq 3$ are not small enough against the R^2 -term. In fact, the possibility of destabilizing the Starobinsky inflationary scenario by the terms with higher powers of the scalar curvature, in the context of $f(R)$ gravity, was noticed earlier in refs. (57; 58). The most general Ansatz (84), which is merely *quadratic* in the supercurvature, does not help for that purpose either.

For example, the full $f(R)$ -gravity function $f_-(R)$ in eq. (97), which we derived from our \mathcal{R}^2 -supergravity, gives rise to the inflaton scalar potential

$$V(y) = V_0 (11e^y + 3) (e^{-y} - 1)^2 \tag{116}$$

where $V_0 = (3^3/2^6)M_{\text{Pl}}^4/f_2^2$. The corresponding inflationary parameters

$$\epsilon(y) = \frac{1}{3} \left[\frac{e^y (11 + 11e^{-y} + 6e^{-2y})}{(11e^y + 3)(e^{-y} - 1)} \right]^2 \geq \frac{1}{3} \tag{117}$$

and

$$\eta(y) = \frac{2}{3} \frac{(11e^y + 5e^{-y} + 12e^{-2y})}{(11e^y + 3)(e^{-y} - 1)^2} \geq \frac{2}{3} \tag{118}$$

are not small enough for matching the WMAP observational data. A solution to this problem is given in the next Sec. 10.

10. Chaotic inflation in $F(\mathcal{R})$ supergravity

Let us take now one more step further and consider a new Ansatz for $F(\mathcal{R})$ function in the cubic form

$$F(\mathcal{R}) = -\frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 - \frac{1}{6}f_3\mathcal{R}^3 \tag{119}$$

whose real (positive) coupling constants $f_{1,2,3}$ are of (mass) dimension 2, 1 and 0, respectively. Our conditions on the coefficients are

$$f_3 \gg 1, \quad f_2^2 \gg f_1 \tag{120}$$

The first condition is needed to have inflation at the curvatures much less than M_{Pl}^2 (and to meet observations), while the second condition is needed to have the scalaron (inflaton) mass be much less than M_{Pl} , in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time.

The bosonic action is given by eq. (72). In the case of a real scalaron it reduces to

$$L/\sqrt{-g} = 2F' \left[\frac{1}{3}R + 4X^2 \right] + 6XF \tag{121}$$

so that the real auxiliary field is a solution to the algebraic equation

$$3F + 11F'X + F'' \left[\frac{1}{3}R + 4X^2 \right] = 0 \tag{122}$$

Stability of the bosonic embedding in supergravity requires $F'(X) < 0$ (Sec. 9). In the case (119) it gives rise to the condition $f_2^2 < f_1f_3$. For simplicity here, we will assume a stronger condition,

$$f_2^2 \ll f_1f_3 \tag{123}$$

Then the second term on the right-hand-side of eq. (119) will not affect inflation, as is shown below.

Equation (121) with the Ansatz (119) reads

$$L = -5f_3X^4 + 11f_2X^3 - (7f_1 + \frac{1}{3}f_3R)X^2 + \frac{2}{3}f_2RX - \frac{1}{3}f_1R \tag{124}$$

and gives rise to a cubic equation on X ,

$$X^3 - \left(\frac{33f_2}{20f_3}\right) X^2 + \left(\frac{7f_1}{10f_3} + \frac{1}{30}R\right) X - \frac{f_2}{30f_3}R = 0 \tag{125}$$

We find three consecutive (overlapping) regimes.

- The high curvature regime including inflation is given by

$$\delta R < 0 \quad \text{and} \quad \frac{|\delta R|}{R_0} \gg \left(\frac{f_2^2}{f_1f_3}\right)^{1/3} \tag{126}$$

where we have introduced the notation $R_0 = 21f_1/f_3 > 0$ and $\delta R = R + R_0$. With our sign conventions we have $R < 0$ during the de Sitter and matter dominated stages. In the regime (126) the f_2 -dependent terms in eqs. (124) and (125) can be neglected, and we get

$$X^2 = -\frac{1}{30}\delta R \tag{127}$$

and

$$L = -\frac{f_1}{3}R + \frac{f_3}{180}(R + R_0)^2 \tag{128}$$

It closely reproduces the Starobinsky inflationary model (Sec. 2) since inflation occurs at $|R| \gg R_0$. In particular, we can identify

$$f_3 = \frac{15M_{\text{Pl}}^2}{M_{\text{inf}}^2} \tag{129}$$

It is worth mentioning that we cannot simply set $f_2 = 0$ in eq. (119) because it would imply $X = 0$ and $L = -\frac{f_1}{3}R$ for $\delta R > 0$. As a result of that the scalar degree of freedom would disappear that would lead to the breaking of a regular Cauchy evolution. Therefore, the second term in eq. (119) is needed to remove that degeneracy.

- The intermediate (post-inflationary) regime is given by

$$\frac{|\delta R|}{R_0} \ll 1 \tag{130}$$

In this case X is given by a root of the cubic equation

$$30X^3 + (\delta R)X + \frac{f_2R_0}{f_3} = 0 \tag{131}$$

It also implies that the 2nd term in eq. (125) is always small. Equation (131) reduces to eq. (127) under the conditions (126).

- The low-curvature regime (up to $R = 0$) is given by

$$\delta R > 0 \quad \text{and} \quad \frac{\delta R}{R_0} \gg \left(\frac{f_2^2}{f_1f_3}\right)^{1/3} \tag{132}$$

It yields

$$X = \frac{f_2R}{f_3(R + R_0)} \tag{133}$$

and

$$L = -\frac{f_1}{3}R + \frac{f_2^2 R^2}{3f_3(R + R_0)} \tag{134}$$

It is now clear that f_1 should be equal to $3M_{pl}^2/2$ in order to obtain the correctly normalized Einstein gravity at $|R| \ll R_0$. In this regime the scalaron mass squared is given by

$$\frac{1}{3|f''(R)|} = \frac{f_3 R_0 M_{pl}^2}{4f_2^2} = \frac{21f_1}{4f_2^2} M_{pl}^2 = \frac{63M_{pl}^4}{8f_2^2} \tag{135}$$

in agreement with the case of the absence of the \mathcal{R}^3 term, studied in the previous section. The scalaron mass squared (135) is much less than M_{pl}^2 indeed, due to the second inequality in eq. (120), but it is much more than one at the end of inflation ($\sim M^2$).

It is worth noticing that the corrections to the Einstein action in eqs. (128) and (134) are of the same order (and small) at the borders of the intermediate region (130).

The roots of the cubic equation (125) are given by the textbook (Cardano) formula (59), though that formula is not very illuminating in a generic case. The Cardano formula greatly simplifies in the most interesting (high curvature) regime where inflation takes place, and the Cardano discriminant is

$$D \approx \left(\frac{R_0}{90}\right)^3 < 0 \tag{136}$$

It implies that all three roots are real and unequal. The Cardano formula yields the roots

$$X_{1,2,3} \approx \frac{2}{3} \sqrt{\frac{-R}{10}} \cos\left(\frac{27}{4f_3 \sqrt{-10R/f_2^2}} + C_{1,2,3}\right) + \frac{11f_2}{20f_3} \tag{137}$$

where the constant $C_{1,2,3}$ takes the values $(\pi/6, 5\pi/6, 3\pi/2)$.

As regards the leading terms, eqs. (124) and (137) result in the $(-R)^{3/2}$ correction to the $(R + R^2)$ -terms in the effective Lagrangian in the high-curvature regime $|R| \gg f_2^2/f_3^2$. In order to verify that this correction does not change our results under the conditions (126), let us consider the $f(R)$ -gravity model with

$$\tilde{f}(R) = R - b(-R)^{3/2} - aR^2 \tag{138}$$

whose parameters $a > 0$ and $b > 0$ are subject to the conditions $a \gg 1$ and $b/a^2 \ll 1$. It is easy to check that $\tilde{f}'(R) > 0$ for $R \in (-\infty, 0]$, as is needed for (classical) stability.

Any $f(R)$ gravity model is classically equivalent to the scalar-tensor gravity with certain scalar potential (Sec. 3). The scalar potential can be calculated from a given function $f(R)$ along the standard lines (Sec. 3). We find (in the high curvature regime)

$$V(y) = \frac{1}{8a} (1 - e^{-y})^2 + \frac{b}{8\sqrt{2a}} e^{-2y} (e^y - 1)^{3/2} \tag{139}$$

in terms of the inflaton field y . The first term of this equation is the scalar potential associated with the pure $(R + R^2)$ model, and the 2nd term is the correction due to the $R^{3/2}$ -term in eq. (138). It is now clear that for large positive y the vacuum energy in the first term dominates and drives inflation until the vacuum energy is compensated by the y -dependent terms near $e^y = 1$.

It can be verified along the lines of ref. (33) that the formula for scalar perturbations remains the same as that for the model (7), ie. $\Delta_{\mathcal{R}}^2 \approx N^2 M_{\text{inf}}^2 / (24\pi^2 M_{\text{Pl}}^2)$, where N is the number of e-folds from the end of inflation. So, to fit the observational data, one has to choose

$$f_3 \approx 5N_e^2 / (8\pi^2 \Delta_{\mathcal{R}}^2) \approx 6.5 \cdot 10^{10} (N_e/50)^2 \quad (140)$$

Here the value of $\Delta_{\mathcal{R}}$ is taken from ref. (14) and the subscript \mathcal{R} has a different meaning from the rest of this paper.

We conclude that the model (119) with a sufficiently small f_2 obeying the conditions (120) and (123) gives a viable realization of the chaotic $(R + R^2)$ -type inflation in supergravity. The only significant difference with respect to the original $(R + R^2)$ inflationary model is the scalaron mass that becomes much larger than M in supergravity, soon after the end of inflation when δR becomes positive. However, it only makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of \mathcal{R} may also be considered, instead of the limited Ansatz (119). The only necessary condition for embedding inflation is that f_3 should be anomalously large. When the curvature grows, the \mathcal{R}^3 -term should become important much earlier than the convergence radius of the whole series without that term. Of course, it means that viable inflation may not occur for any function $F(\mathcal{R})$ but only inside a small region of non-zero measure in the space of all those functions. However, the same is true for all known inflationary models, so the very existence of inflation has to be taken from the observational data, not from a pure thought.

The results of this Section can be considered as the viable alternative to the earlier fundamental proposals (49; 50) for realization of chaotic inflation in supergravity. But inflation is not the only target of our construction. As is well known (20; 21; 60), the scalaron decays into pairs of particles and anti-particles of quantum matter fields, while its decay into gravitons is strongly suppressed (61). It thus represents the *universal mechanism of viable reheating* after inflation and provides a transition to the subsequent hot radiation-dominated stage of the Universe evolution. In its turn, it leads to the standard primordial nucleosynthesis (BBN) after. In $F(R)$ supergravity the scalaron has a pseudo-scalar superpartner (axion) that may be the source of a strong CP -violation and then, subsequently, lepto- and baryo-genesis that naturally lead to baryon (matter-antimatter) asymmetry (65; 66) — see Secs. 12 and 14 for more.

11. Nonminimal scalar-curvature coupling in gravity and supergravity

It was recently proposed in refs. (67; 68; 70) to identify Higgs scalar with inflaton, by employing a nonminimal coupling of the Higgs scalar to the scalar curvature of spacetime. Adding such nonminimal coupling to gravity is natural in curved spacetime because it is required by renormalization (71).

Let us compare the inflationary scalar potential, derived by the use of the nonminimal coupling (67; 68; 70), with the scalar potential that follows from the $(R + R^2)$ inflationary model (Sec. 2), and confirm their equivalence. In what follows we will upgrade that equivalence to supergravity. In this section we set $M_{\text{Pl}} = 1$ too.

The original motivation of refs. (67; 68; 70) was based on the assumption that there is no new physics beyond the Standard Model up to the Planck scale. Then it is natural to search for the Higgs mechanism of inflation by identifying inflaton with Higgs. Our motivation here is different: we assume that there is the new physics beyond the Standard Model, and it is given by supersymmetry. Then it is quite natural to search for most economical mechanisms

of inflation in the context of supergravity. Moreover, we do not have to identify inflaton with a Higgs particle of the Minimal Supersymmetric Standard Model. Let us begin with the 4D Lagrangian

$$\mathcal{L}_J = \sqrt{-g_J} \left\{ -\frac{1}{2}(1 + \xi\phi_J^2)R_J + \frac{1}{2}g_J^{\mu\nu}\partial_\mu\phi_J\partial_\nu\phi_J - V(\phi_J) \right\} \tag{141}$$

where we have introduced the real scalar field $\phi_J(x)$, nonminimally coupled to gravity (with the coupling constant ξ) in Jordan frame, with the Higgs-like scalar potential

$$V(\phi_J) = \frac{\lambda}{4}(\phi_J^2 - v^2)^2 \tag{142}$$

The action (141) can be rewritten to Einstein frame by redefining the metric via a Weyl transformation,

$$g^{\mu\nu} = \frac{g_J^{\mu\nu}}{(1 + \xi\phi_J^2)} \tag{143}$$

It gives rise to the standard Einstein-Hilbert term ($-\frac{1}{2}R$) for gravity in the Lagrangian. However, it also leads to a nonminimal (or noncanonical) kinetic term of the scalar field ϕ_J . To get the canonical kinetic term, a scalar field redefinition is needed, $\phi_J \rightarrow \varphi(\phi_J)$, subject to the condition

$$\frac{d\varphi}{d\phi_J} = \frac{\sqrt{1 + \xi(1 + 6\xi)\phi_J^2}}{1 + \xi\phi_J^2} \tag{144}$$

As a result, the non-minimal theory (141) is classically equivalent to the standard (canonical) theory of the scalar field $\varphi(x)$ minimally coupled to gravity,

$$\mathcal{L}_E = \sqrt{-g} \left\{ -\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) \right\} \tag{145}$$

with the scalar potential

$$V(\varphi) = \frac{V(\phi_J(\varphi))}{[1 + \xi\phi_J^2(\varphi)]^2} \tag{146}$$

Given a large positive $\xi \gg 1$, in the small field limit one finds from eq. (144) that $\phi_J \approx \varphi$, whereas in the large φ limit one gets

$$\varphi \approx \sqrt{\frac{3}{2}} \log(1 + \xi\phi_J^2) \tag{147}$$

Then eq. (146) yields the scalar potential:

(i) in the *very small* field limit, $\varphi < \sqrt{\frac{2}{3\xi}}\xi^{-1}$, as

$$V_{vs}(\varphi) \approx \frac{\lambda}{4}\varphi^4 \tag{148}$$

(ii) in the *small* field limit, $\sqrt{\frac{2}{3}}\xi^{-1} < \varphi \ll \sqrt{\frac{3}{2}}$, as

$$V_s(\varphi) \approx \frac{\lambda}{6\xi^2}\varphi^2, \tag{149}$$

(iii) and in the *large* field limit, $\varphi \gg \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V(\varphi) \approx \frac{\lambda}{4\xi^2} \left(1 - \exp \left[-\sqrt{\frac{2}{3}}\varphi \right] \right)^2 \tag{150}$$

We have assumed here that $\xi \gg 1$ and $v\xi \ll 1$.

Identifying inflaton with Higgs particle requires the parameter v to be of the order of weak scale, and the coupling λ to be the Higgs boson selfcoupling at the inflationary scale. The scalar potential (150) is perfectly suitable to support a slow-roll inflation, while its consistency with the COBE normalization condition (Sec. 4) for the observed CMB amplitude of density perturbations (eg., at the e-foldings number $N_e = 50 \div 60$) gives rise to the relation $\xi/\sqrt{\lambda} \approx \mathcal{O}(10^5)$ (67; 68; 70).

The scalar potential (149) corresponds to the post-inflationary matter-dominated epoch described by the oscillating inflaton field φ with the frequency

$$\omega = \sqrt{\frac{\lambda}{3}}\xi^{-1} = M_{\text{inf}} \tag{151}$$

When gravity is extended to 4D, $N = 1$ supergravity, any physical real scalar field should be complexified, becoming the leading complex scalar field component of a chiral (scalar) matter supermultiplet. In a curved superspace of $N = 1$ supergravity, the chiral matter supermultiplet is described by a covariantly chiral superfield Φ obeying the constraint $\overline{\nabla}_{\dot{\alpha}}\Phi = 0$. The standard (generic and minimally coupled) matter-supergravity action is given by in superspace by eqs. (44) and (46), namely,

$$S_{\text{MSG}} = -3 \int d^4x d^4\theta E^{-1} \exp \left[-\frac{1}{3}K(\Phi, \overline{\Phi}) \right] + \left\{ \int d^4x d^2\theta \mathcal{E}W(\Phi) + \text{H.c.} \right\} \tag{152}$$

in terms of the Kähler potential $K = -3 \log(-\frac{1}{3}\Omega)$ and the superpotential W of the chiral supermatter, and the full density E and the chiral density \mathcal{E} of the superspace supergravity. The non-minimal matter-supergravity coupling in superspace reads

$$S_{\text{NM}} = \int d^4x d^2\theta \mathcal{E}X(\Phi)\mathcal{R} + \text{H.c.} \tag{153}$$

in terms of the chiral function $X(\Phi)$ and the $N=1$ chiral scalar supercurvature superfield \mathcal{R} obeying $\overline{\nabla}_{\dot{\alpha}}\mathcal{R} = 0$. In terms of the field components of the superfields the non-minimal action (153) is given by

$$\int d^4x d^2\theta \mathcal{E}X(\Phi)\mathcal{R} + \text{H.c.} = -\frac{1}{6} \int d^4x \sqrt{-g}X(\phi_c)R + \text{H.c.} + \dots \tag{154}$$

stand for the fermionic terms, and $\phi_c = \Phi| = \phi + i\chi$ is the leading complex scalar field component of the superfield Φ . Given $X(\Phi) = -\xi\Phi^2$ with the real coupling constant ξ , we find the bosonic contribution

$$S_{\text{NM,bos.}} = \frac{1}{6}\xi \int d^4x \sqrt{-g} (\phi^2 - \chi^2) R \tag{155}$$

It is worth noticing that the supersymmetrizable (bosonic) non-minimal coupling reads $[\phi_c^2 + (\phi_c^\dagger)^2] R$, not $(\phi_c^\dagger\phi_c)R$.

Let us now introduce the manifestly supersymmetric nonminimal action (in Jordan frame) as

$$S = S_{\text{MSG}} + S_{\text{NM}} \tag{156}$$

In curved superspace of $N = 1$ supergravity the (Siegel’s) chiral integration rule

$$\int d^4x d^2\theta \mathcal{E} \mathcal{L}_{\text{ch}} = \int d^4x d^4\theta E^{-1} \frac{\mathcal{L}_{\text{ch}}}{\mathcal{R}} \tag{157}$$

applies to any chiral superfield Lagrangian \mathcal{L}_{ch} with $\bar{\nabla}_{\dot{\alpha}} \mathcal{L}_{\text{ch}} = 0$. It is, therefore, possible to rewrite eq. (153) to the equivalent form

$$S_{\text{NM}} = \int d^4x d^4\theta E^{-1} [X(\Phi) + \bar{X}(\bar{\Phi})] \tag{158}$$

We conclude that adding S_{NM} to S_{MSG} is equivalent to the simple change of the Ω -potential as (cf. ref. (72))

$$\Omega \rightarrow \Omega_{\text{NM}} = \Omega + X(\Phi) + \bar{X}(\bar{\Phi}) \tag{159}$$

It amounts to the change of the Kähler potential as

$$K_{\text{NM}} = -3 \ln \left[e^{-K/3} - \frac{X(\Phi) + \bar{X}(\bar{\Phi})}{3} \right] \tag{160}$$

The scalar potential in the matter-coupled supergravity (152) is given by eq. (56),

$$V(\phi, \bar{\phi}) = e^G \left[\left(\frac{\partial^2 G}{\partial \phi \partial \bar{\phi}} \right)^{-1} \frac{\partial G}{\partial \phi} \frac{\partial G}{\partial \bar{\phi}} - 3 \right] \tag{161}$$

in terms of the Kähler-gauge-invariant function (52), ie.

$$G = K + \ln |W|^2 \tag{162}$$

Hence, in the nonminimal case (156) we have

$$G_{\text{NM}} = K_{\text{NM}} + \ln |W|^2 \tag{163}$$

Contrary to the bosonic case, one gets a nontrivial Kähler potential K_{NM} , ie. a *Non-Linear Sigma-Model* (NLSM) as the kinetic term of $\phi_c = \phi + i\chi$ (see ref. (48) for more about the NLSM). Since the NLSM target space in general has a nonvanishing curvature, no field redefinition generically exist that could bring the kinetic term to the free (canonical) form with its Kähler potential $K_{\text{free}} = \bar{\Phi}\Phi$.

Let's now consider the full action (156) under the slow-roll condition, ie. when the contribution of the kinetic term is negligible. Then eq. (156) takes the truly chiral form

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} [X(\Phi)\mathcal{R} + W(\Phi)] + \text{H.c.} \quad (164)$$

When choosing X as the independent chiral superfield, $S_{\text{ch.}}$ can be rewritten to the form

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} [X\mathcal{R} - \mathcal{Z}(X)] + \text{H.c.} \quad (165)$$

where we have introduced the notation

$$\mathcal{Z}(X) = -W(\Phi(X)) \quad (166)$$

In its turn, the action (165) is equivalent to the chiral $F(\mathcal{R})$ supergravity action (34), whose function F is related to the function \mathcal{Z} via Legendre transformation (Sec. 6)

$$\mathcal{Z} = X\mathcal{R} - F, \quad F'(\mathcal{R}) = X \quad \text{and} \quad \mathcal{Z}'(X) = \mathcal{R} \quad (167)$$

It implies the equivalence between the reduced action (164) and the corresponding $F(\mathcal{R})$ supergravity whose F -function obeys eq. (167).

Next, let us consider the special case of eq. (164) when the superpotential is given by

$$W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\Phi^3 \quad (168)$$

with the real coupling constants $m > 0$ and $\lambda > 0$. The model (168) is known as the *Wess-Zumino* (WZ) model in 4D, $N = 1$ rigid supersymmetry. It has the most general renormalizable scalar superpotential in the absence of supergravity. In terms of the field components, it gives rise to the Higgs-like scalar potential.

For simplicity, let us take a cubic superpotential,

$$W_3(\Phi) = \frac{1}{6}\lambda\Phi^3 \quad (169)$$

or just assume that this term dominates in the superpotential (168), and choose the $X(\Phi)$ -function in eq. (164) in the form

$$X(\Phi) = -\zeta\Phi^2 \quad (170)$$

with a large positive coefficient ζ , $\zeta > 0$ and $\zeta \gg 1$, in accordance with eqs. (154) and (155).

Let us also simplify the F -function of eq. (119) by keeping only the most relevant cubic term,

$$F_3(\mathcal{R}) = -\frac{1}{6}f_3\mathcal{R}^3 \quad (171)$$

It is straightforward to calculate the \mathcal{Z} -function for the F -function (171) by using eq. (167). We find

$$-X = \frac{1}{2}f_3\mathcal{R}^2 \quad \text{and} \quad \mathcal{Z}'(X) = \sqrt{\frac{-2X}{f_3}} \quad (172)$$

Integrating the last equation with respect to X yields

$$\mathcal{Z}(X) = -\frac{2}{3}\sqrt{\frac{2}{f_3}}(-X)^{3/2} = -\frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{173}$$

where we have used eq. (170). In accordance to eq. (166), the $F(\mathcal{R})$ -supergravity \mathcal{Z} -potential (173) implies the superpotential

$$W_{\text{KS}}(\Phi) = \frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{174}$$

It coincides with the superpotential (169) of the WZ-model, provided that we identify the couplings as

$$f_3 = \frac{32\xi^3}{\lambda^2} \tag{175}$$

We conclude that the original nonminimally coupled matter-supergravity theory (156) in the slow-roll approximation with the superpotential (169) is classically equivalent to the $F(\mathcal{R})$ -supergravity theory with the F -function given by eq. (171) when the couplings are related by eq. (175).

The inflaton mass M in the supersymmetric case, according to eqs. (129) and (175), is given by

$$M_{\text{inf}}^2 = \frac{15\lambda^2}{32\xi^3} \tag{176}$$

This relation is different from eq. (151) valid in the bosonic case. Since the value of M_{inf} is fixed by the COBE normalization (Sec. 4), eq. (176) implies that the value of ξ is much lower in the supersymmetric case, $\xi \approx \mathcal{O}(10^{10/3})$, where we have used eqs. (140) and (175), and have assumed that $\lambda \approx \mathcal{O}(1)$.

The established equivalence begs for a fundamental reason. In the high-curvature (inflationary) regime the R^2 -term dominates over the R -term in the Starobinsky action (7), while the coupling constant in front of the R^2 -action is dimensionless (Sect. 2). The Higgs inflation is based on the Lagrangian (141) with the relevant scalar potential $V_4 = \frac{1}{4}\lambda\phi_J^4$ (the parameter v is irrelevant for inflation), whose coupling constants ξ and λ are also dimensionless. Therefore, both relevant actions are *conformal*. Inflation breaks the conformal symmetry spontaneously.

The supersymmetric case is similar: the nonminimal action (164) with the X -function (170) and the superpotential (169) also has only dimensionless coupling constants ξ and λ , while the same is true for the $F(\mathcal{R})$ -supergravity action with the F -function (171), whose coupling constant f_3 is dimensionless too. Therefore, those actions are both *superconformal*, while inflation spontaneously breaks the superconformal invariance.

A spontaneous breaking of the conformal symmetry necessarily leads to Goldstone particle (or dilaton) associated with spontaneously broken dilatations. So, perhaps, Starobinsky scalaron (inflaton) may be identified with the Goldstone dilaton!

The basic field theory model, describing both inflation *and* the subsequent reheating, reads (see eg., eq. (6) in ref. (73))

$$\begin{aligned} L/\sqrt{-g} = & \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi \\ & - \frac{1}{2}g^2\phi^2\chi^2 - h(\bar{\psi}\psi)\phi \end{aligned} \tag{177}$$

with the inflaton scalar field ϕ interacting with another scalar field χ and a spinor field ψ . The nonminimal supergravity theory (156) with the Wess-Zumino superpotential (168) can be considered as the $N = 1$ locally supersymmetric extension of the basic model (177), after rescaling ϕ_c to $(1/\sqrt{2})\phi_c$ and identifying $\tilde{\xi} = -\frac{1}{3}\xi$ because of eq. (155). Therefore, *pre-heating* (ie. the nonperturbative enhancement of particle production due to parametric resonance (73)) is a generic feature of supergravity models.

The axion χ and fermion ψ are both required by supersymmetry, being in the same chiral supermultiplet with the inflaton ϕ . The scalar interactions are

$$V_{\text{int}}(\phi, \chi) = m\lambda\phi(\phi^2 + \chi^2) + \frac{\lambda^2}{4}(\phi^2 + \chi^2)^2 \quad (178)$$

whereas the Yukawa couplings are given by

$$L_{\text{Yu}} = \frac{1}{2}\lambda\phi(\bar{\psi}\psi) + \frac{1}{2}\lambda\chi(\bar{\psi}i\gamma_5\psi) \quad (179)$$

Supersymmetry implies the unification of couplings since $h = -\frac{1}{2}\lambda$ and $g^2 = \lambda^2$ in terms of the single coupling constant λ . If supersymmetry is unbroken, the masses of ϕ , χ and ψ are all the same. However, inflation already breaks supersymmetry, so the spontaneously broken supersymmetry is appropriate here.

To conclude, inflationary (slow-roll) dynamics in the gravity theory with a nonminimal scalar-curvature coupling can be equivalent to that in the certain $f(R)$ gravity theory. We extended that correspondence to $N = 1$ supergravity. The nonminimal coupling in supergravity is rewritten in terms of the standard ('minimal') $N = 1$ matter-coupled supergravity, by using their manifestly supersymmetric formulations in curved superspace. The equivalence relation between the supergravity theory with the nonminimal scalar-curvature coupling and the $F(\mathcal{R})$ supergravity during slow-roll inflation is established.

The equivalence is expected to hold even after inflation, during initial reheating with harmonic oscillations. In the bosonic case the equivalence holds until the inflaton field value is higher than $\omega \approx M_{\text{Pl}}/\xi \approx 10^{-5}M_{\text{Pl}}$. In the supersymmetric case we have the same bound $\omega \approx M_{\text{Pl}}/\xi^{3/2} \approx 10^{-5}M_{\text{Pl}}$.

12. Reheating and quantum particle production

Reheating is a transfer of energy from inflaton to ordinary particles and fields.

The classical solution (neglecting particle production) near the minimum of the inflaton scalar potential reads

$$a(t) \approx a_0 \left(\frac{t}{t_0}\right)^{2/3} \quad \text{and} \quad \varphi(t) \approx \left(\frac{M_{\text{Pl}}}{3M_{\text{inf}}}\right) \frac{\cos[M_{\text{inf}}(t - t_0)]}{t - t_0} \quad (180)$$

A *time-dependent* classical spacetime background leads to *quantum* production of particles with masses $m < \omega = M_{\text{inf}}$ (71). Actually, the amplitude of φ -oscillations decreases much faster (73), namely, as

$$\exp\left[-\frac{1}{2}(3H + \Gamma)t\right] \quad (181)$$

via inflaton decay and the universe expansion, as the solution to the inflaton equation

$$\ddot{\varphi} + 3H\dot{\varphi} + (m^2 + \Pi)\varphi = 0 \quad (182)$$

Here Π denotes the polarization operator that effectively describes particle production. Unitarity (optical theorem) requires $\text{Im}(\Pi) = m\Gamma$. The assumption $m \gg H$ has also been used here (73).

The Starobinsky model in Jordan frame,

$$S = \int d^4x \sqrt{-g_J} f_S(R_J) + S_{\text{SM}}(g^{\mu\nu}_J, \psi) \tag{183}$$

after the conformal transformation to Einstein frame reads

$$S = S_{\text{scalar-tensor gravity}}(g_{\mu\nu}, \varphi) + S_{\text{SM}}(g^{\mu\nu} e^{-\sigma\varphi}, \psi) \tag{184}$$

so that the inflaton φ couples to all non-conformal terms and fields ψ , due to the universality of gravitational interaction. Therefore, the Starobinsky inflation also has the *universal* mechanism of particle production.

The perturbative decay rates of inflaton into a pair of scalars (s) or into a pair of fermions (f) are given by (20; 21; 74)

$$\Gamma_{\varphi \rightarrow ss} = \frac{M_{\text{inf}}^3}{192\pi M_{\text{Pl}}^2} \quad \text{and} \quad \Gamma_{\varphi \rightarrow ff} = \frac{M_{\text{inf}} M_f^2}{48\pi M_{\text{Pl}}^2} \tag{185}$$

The perturbative decay rate of inflaton into a pair of *gravitini* is (75)

$$\Gamma_{\varphi \rightarrow 2\psi_{3/2}} = \frac{|G_{,\varphi}|^2}{288\pi} \frac{M_{\text{inf}}^5}{m_{3/2}^2 M_{\text{Pl}}^2} \tag{186}$$

There is *no* parametric resonance enhancement here because the produced particles rapidly scatter. The energy transfers by the time $t_{\text{reh}} \geq (\sum_{s,f} \Gamma_{s,f})^{-1}$. One finds the *reheating temperature* (34; 76)

$$T_{\text{reh}} \propto \sqrt{\frac{M_{\text{Pl}}\Gamma}{(\#d.o.f.)^{1/2}}} \approx 10^9 \text{ GeV} \tag{187}$$

that gives the maximal temperature of the primordial plasma.

In the context of supergravity coupled to the supersymmetric matter (like MSSM) gravitino can be either LSP (= the lightest sparticle) or NLSP (= not LSP). In the LSP case (that usually happens with gauge mediation of supersymmetry breaking and $m_{3/2} \ll 10^2$ GeV) gravitino is stable due to the R-parity conservation. If gravitino is NLSP, then it is unstable (it usually happens with gravity- or anomaly- mediation of supersymmetry breaking, and $m_{3/2} \gg 10^2$ GeV). Unstable gravitino can decay into LSP. See ref. (77) for a review of mediation of supersymmetry breaking from the hidden sector to the visible sector.

Stable gravitino may be the dominant part of *Cold Dark Matter* (CDM) (78). There exist severe Big Bang Nucleosynthesis (BBN) ⁴ constraints on the overproduction of ³He in that case, which give rise to the upper bound on the reheating temperature of thermally produced gravitini, $T_{\text{reh}} < 10^{5-6}$ GeV (69; 80). The reheating temperature (187) is unrelated to that bound because it corresponds to the much earlier time in the history of the Universe.

When gravitino is NLSP of mass $m_{3/2} \gg 10^2$ GeV, the BBN constraints are drastically relaxed because the gravitino lifetime becomes much shorter than the BBN time (about 1 sec) (69;

⁴ See ref. (79) for a review of BBN.

80). In that case the most likely CDM candidate is MSSM neutralino, while the reheating temperature may be as high as 10^{10} GeV (80).

An overproduction of gravitini from inflaton decay and scattering processes should be avoided in order to not overclose the Universe. The cosmological constraints on gravitino abundances were formulated in ref. (81). Those constraints are very model-dependent.

The rate of decay changes with time, along with the decreasing amplitude of inflaton oscillations. It stops when the decay rate becomes smaller than the production rate. Then the particle production accelerates (called *pre-heating*, or true BB!) due to the parametric resonance enhancement (73). The reheating rapidly transfers most of energy to radiation, and leads to a radiation-dominated universe with $a \propto t^{1/2}$.

In the matter-coupled $F(\mathcal{R})$ supergravity with the action

$$S = \left[\int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \right] + S_{\text{SSM}}(E, \Psi) \quad (188)$$

after the super-Weyl transformation, $\mathcal{E} \rightarrow \mathcal{E} e^{3\Phi}$, we get

$$S = S_{\text{scalar-tensor supergravity}}(E, \Phi) + S_{\text{SSM}}(e^{\Phi+\bar{\Phi}}E, \Psi) \quad (189)$$

so that the superscalaron Φ is *universally* coupled to the SSM matter superfields Ψ .

13. Conclusion

- A manifestly 4D, $N = 1$ supersymmetric *extension* of $f(R)$ gravity exist, it is *chiral* and is parametrized by a holomorphic function. An $F(R)$ supergravity is classically *equivalent* to the standard theory of a chiral scalar superfield (with certain Kähler potential and superpotential) minimally coupled to the $N = 1$ Poincaré supergravity in four spacetime dimensions (with nontrivial G and K).

- The Starobinsky model of chaotic inflation can be embedded into $F(\mathcal{R})$ supergravity, thus providing the new viable realization of chaotic inflation in supergravity, and the simple solution to the η -problem, by using supergravity only!

The dynamical chiral superfield in $F(\mathcal{R})$ supergravity may be identified with the dilaton-axion chiral superfield in quantum 4D Superstring Theory, when demanding the $SL(2, \mathbf{Z})$ symmetry of the effective action. The $R^2 A(R)$ terms may appear in the bosonic gravitational effective action after superstring compactification. The problem is to get the anomalously *large* coefficient in front of the \mathcal{R}^3 -term in the effective $F(\mathcal{R})$ supergravity theory, that would be consistent with string dynamics.

Supersymmetry in $F(R)$ supergravity is already broken by inflation. The anomaly- or gravitationally-mediated supersymmetry breaking may serve as the important element for the new particle phenomenology (beyond the Standard Model) based on the matter-coupled $F(R)$ supergravity theory.

14. Outlook: CP -violation, baryonic asymmetry, lepto- and baryo-genesis

The observed part of our Universe is C - and CP -asymmetric, and it has no antimatter. Inflation naturally implies a *dynamical* origin of the baryonic matter predominance due to a nonconserved baryon number. The main conditions for the dynamical generation of the cosmological baryon asymmetry in the early universe were formulated by A.D. Sakharov in 1967 (62):

1. nonconservation of baryons (*cf.* SUSY, GUT, EW theory),
2. *C*– and *CP*–symmetry breaking (confirmed experimentally),
3. deviation from thermal equilibrium in initial hot universe.

There exist many scenarios of baryogenesis (see ref. (63) for a review), all designed to explain the observed asymmetry (BBN,CMB):

$$\beta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6 \cdot 10^{-10} \tag{190}$$

Here n_B stands for the concentration of baryons, $n_{\bar{B}}$ for the concentration of anti-baryons, and n_γ for the concentration of photons.

Perhaps, the most popular scenario is the nonthermal *baryo-through-lepto-genesis* (64; 65), ie. a creation of lepton asymmetry by *L*-nonconserving decays of a heavy ($m \approx 10^{10}$ GeV) Majorana neutrino, and a subsequent transformation of the lepton asymmetry into baryonic asymmetry by *CP*-symmetric, *B*-nonconserving and (*B-L*)-conserving electro-weak processes.

The thermal leptogenesis requires high reheating temperature, $T_{\text{reh}} \geq 10^9$ GeV (82), which is consistent with our eq. (187).

The matter-coupled *F(R)* supergravity theory may contribute towards the *origin* and the *mechanism* of *CP*-violation and baryon asymmetry, because

- complex coefficients of *F(R)*-function and the complex nature of the *F(R)* supergravity are the simple *source* of explicit *CP*-violation and complex Yukawa couplings;
- the nonthermal leptogenesis is possible via decay of heavy sterile neutrinos (*FY*-mechanism) *universally produced* by (super)scalaron decays, or via neutrino oscillations in early universe (83);
- the existence of the natural Cold Dark Matter candidates (*gravitino, axion, inflatino*) in *F(R)* supergravity;
- *F(R)*-supergravity can naturally support *hybrid* (or two-field) inflationary models because it already has a pseudo-scalar superpartner (axion) of inflaton. As is well known, *non-Gaussianity* is a measure of inflaton *interactions* (determined by its 3-point functions and higher). The non-Gaussianity parameter f_{NL} is defined in terms of the (gauge-invariant) comoving curvature perturbations as

$$\hat{\mathcal{R}} = \mathcal{R}_{\text{gr}} + \frac{3}{5} f_{\text{NL}} \mathcal{R}_{\text{gr}}^2 \tag{191}$$

The non-Gaussianity was not observed yet, though it is expected, while all single-field inflationary models predict $f_{\text{NL}} \approx 0.02$.

15. Acknowledgements

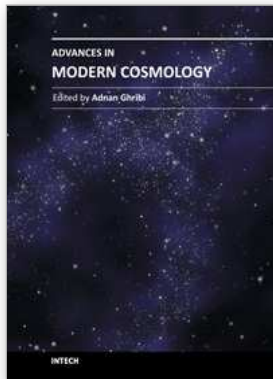
I am grateful to my collaborators, S.J. Gates Jr., A.A. Starobinsky and N. Yunes, and my students, S. Kaneda and N. Watanabe, for their efforts. I wish to thank the Theory Division of CERN in Geneva, the Heisenberg Institute of Physics in Munich, the Einstein Institute of Gravitational Physics in Potsdam and the Center for Theoretical Physics in Marseille for kind hospitality extended to me during preparation of this paper.

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Advances in Modern Cosmology

Edited by Dr. Adnan Ghribi

ISBN 978-953-307-423-8

Hard cover, 198 pages

Publisher InTech

Published online 29, August, 2011

Published in print edition August, 2011

The twentieth century elevated our understanding of the Universe from its early stages to what it is today and what is to become of it. Cosmology is the weapon that utilizes all the scientific tools that we have created to feel less lost in the immensity of our Universe. The standard model is the theory that explains the best what we observe. Even with all the successes that this theory had, two main questions are still to be answered: What is the nature of dark matter and dark energy? This book attempts to understand these questions while giving some of the most promising advances in modern cosmology.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Sergei V. Ketov (2011). F(R) Supergravity and Early Universe: the Meeting Point of Cosmology and High-Energy Physics, *Advances in Modern Cosmology*, Dr. Adnan Ghribi (Ed.), ISBN: 978-953-307-423-8, InTech, Available from: <http://www.intechopen.com/books/advances-in-modern-cosmology/f-r-supergravity-and-early-universe-the-meeting-point-of-cosmology-and-high-energy-physics>

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