Chapter

Parameter Estimation of Weighted Maxwell-Boltzmann Distribution Using Simulated and Real Life Data Sets

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Abstract

This paper deals with estimation of parameters of Weighted Maxwell-Boltzmann Distribution by using Classical and Bayesian Paradigm. Under Classical Approach, we have estimated the rate parameter using Maximum likelihood Estimator. In Bayesian Paradigm, we have primarily studied the Bayes' estimator of the parameter of the Weighted Maxwell-Boltzmann Distribution under the extended Jeffrey's prior, Gamma and exponential prior distributions assuming different loss functions. The extended Jeffrey's prior gives the opportunity of covering wide spectrum of priors to get Bayes' estimates of the parameter – particular cases of which are Jeffrey's prior and Hartigan's prior. A comparative study has been done between the MLE and the estimates of different loss functions (SELF and Al-Bayyati's, Stein and Precautionary new loss function). From the results, we observe that in most cases, Bayesian Estimator under New Loss function (Al-Bayyati's Loss function) has the smallest Mean Squared Error values for both prior's i.e., Jeffrey's and an extension of Jeffrey's prior information. Moreover, when the sample size increases, the MSE decreases quite significantly. These estimators are then compared in terms of mean square error (MSE) which is computed by using the programming language R. Also, two types of real life data sets are considered for making the model comparison between special cases of Weighted Maxwell-Boltzmann Distribution in terms of fitting.

Keywords: Weighted Maxwell-Boltzmann Distribution, prior distributions, loss functions, R Software

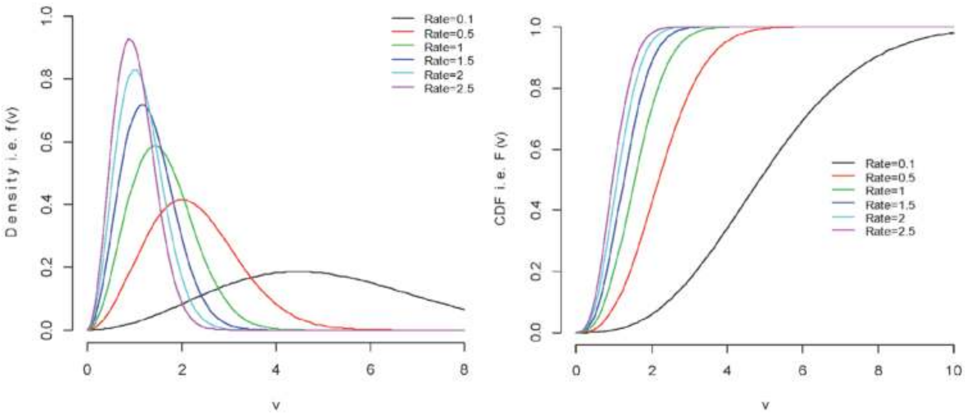
1. Introduction

In Statistical Mechanics, there are a lot of applications of Maxwell-Boltzmann Distribution. The Maxwell-Boltzmann distribution forms the basis of the kinetic energy of gases, which explains many fundamental properties of gases, including pressure and diffusion. This distribution is sometimes called as the distribution of velocities, energy and magnitude of momenta of molecules. Tyagi and Bhattacharya [1] who considered the Maxwell distribution as a lifetime model and discussed the Baye's and minimum variance unbiased estimation procedures for its parameter and reliability function. Chaturvedi and Rani [2] estimated the classical and Baye's estimators for the Maxwell distribution, after generalization it by adding another parameter. Empirical Baye's estimation for the Maxwell distribution was also obtained by Bekker and Roux [3]. Kazmi et al. [4] derived the Bayesian estimation for two component mixture of Maxwell distribution, assuming censored data. The Maxwell-Boltzmann distribution can be used to find the distribution of particle's kinetic energy which is related to particle's speed by the formula $E = mv^2/2$, provided the distribution of speed is known. The PDF of Maxwell-Boltzmann distribution is given by Maxwell [5]:

$$f_w(x) = \sqrt{(2/\pi)}\,\theta^{3/2} x^2 e^{-\theta\, x^2/2} \tag{1}$$

And the CDF of Maxwell Distribution is given as:

$$F_w(x) = 1 - \frac{\Gamma\left((\alpha+3)/2, \theta x^2/2\right)}{\Gamma((\alpha+3)/2)} \tag{2}$$



Recently, Aijaz et al. [6] estimates and analyze the Bayes' Estimators of Maxwell-Boltzmann Distribution under various Loss functions and prior Distributions. Other Contributions in Maxwell Distribution are Huang and Chen [7], Krishna and Malik [8], Tomer and Panwar [9], Zhang et al. [10], and Monisa [11].

Various Statisticians and Mathematicians have carried out the Bayesian paradigm of Maxwell-Boltzmann distribution by using loss functions and prior distributions, see Al-Baldawi [12], Dey et al. [13], Podder and Roy [14], Rasheed [15], and Spiring and Yeung [16].

The concept of weighted distributions introduced by Fisher [17] and later it was formulated in general terms by Rao [18] in connection with modeling statistical data. These Distributions are applicable, when each and every observation is given an equal chance of being recorded. These distributions arise, when the probability of selecting an observation varied from observation to observation. In this context, the authors generalize the Maxwell Distribution and is known as Weighted Maxwell-Boltzmann distribution. The PDF of Weighted Maxwell-Boltzmann Distribution was introduced by Aijaz et al. [19].

$$f_w(x;\theta,\alpha) = \frac{\theta^{(\alpha+3)/2} x^{(\alpha+2)} e^{-\theta\, x^2/2}}{2^{(\alpha+1)/2}\Gamma((\alpha+3)/2)} \tag{3}$$

Where θ is the rate parameter and ω is the weight parameter (ω>0).

Also, CDF of the Weighted Maxwell Distribution is given by:

$$F_w(x) = 1 - \frac{\Gamma\left((\alpha+3)/2, \theta x^2/2\right)}{\Gamma((\alpha+3)/2)} \tag{4}$$

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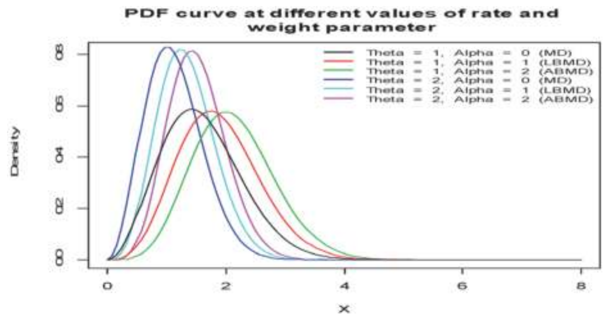
The Reliability function and Hazard Rate of the Weighted Maxwell Distribution is given by:

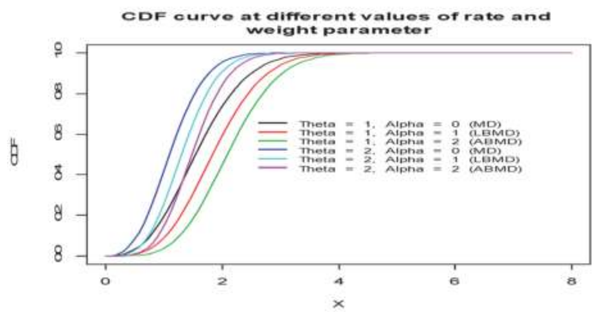
$$R_w(x) = \frac{\Gamma\left((\alpha+3)/2, \theta x^2/2\right)}{\Gamma((\alpha+3)/2)} \tag{5}$$

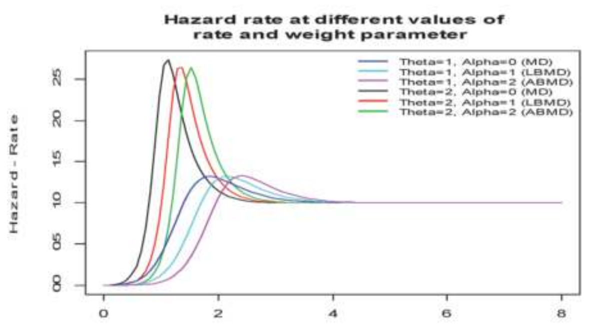
$$h_w(x) = \frac{\theta^{((\alpha+3)/2)} x^{(\alpha+2)} \exp\left(-\theta x^2/2\right)}{2^{((\alpha+1)/2)} \Gamma((\alpha+3)/2) \Gamma\left((\alpha+3)/2, \theta x^2/2\right)} \tag{6}$$

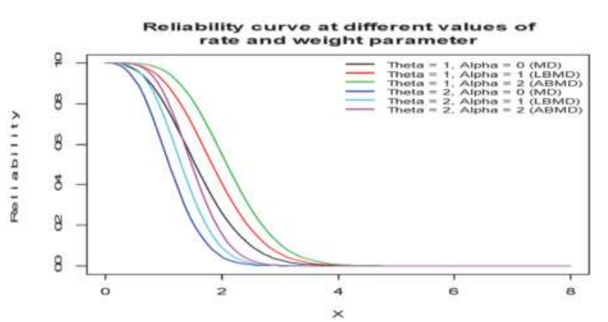
The, r^{th} moments about zero of Weighted Maxwell-Boltzmann Distribution is given by:

$$\mu_r' = (2/\theta)^{\frac{r}{2}} \Gamma((\alpha+r+3)/2)/\Gamma((\alpha+3)/2), \text{Where } r = 1, 2, 3, 4, \ldots \tag{7}$$









In comparison to classical approach, Bayesian approach is considered to be fair enough in estimating the parameters of a distribution provided that the prior distribution describes nicely the random behavior of a parameter. Very often, priors are chosen according to one's subjective knowledge and beliefs that is why Bayesian approach is sometimes called as subjective approach. However, Aslam [20] have shown an application of prior predictive distribution to elicit the prior density. A number of symmetric and asymmetric loss functions have been shown to be functional, see Kasair et al. [21], Norstrom [22], Reshi et al. [23], Zellner [24], Reshi et al. [25], Dey and Maiti [26], Alkutbi [27], Wald [28], etc.

2. Estimation of parameters

In this Section, the authors estimated the parameters of Weighted Maxwell-Boltzmann Distribution under Classical and Bayesian Paradigm.

2.1 Maximum likelihood estimation

Let $x = (x_1, x_2, x_3, \dots, x_n)$ be a random sample of size n from Weighted Maxwell Distribution Therefore the likelihood function will be given by:

$$L(\theta, \omega/x) = \frac{\theta^{n\left(\frac{\omega+3}{2}\right)} \sum {x_i}^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\omega+3}{2}\right)^n} \exp\left(-\frac{\theta}{2} \sum\nolimits_{i=1}^{n} x_{i^2}\right) \tag{8}$$

The, the Log likelihood function is given by:

$$\begin{aligned} \log L(\theta, \omega/x) = {} & \frac{n(\omega+3)}{2} \log(\theta) - \frac{n(\omega+1)}{2} \log(2) - n \log \Gamma\left(\frac{\omega+3}{2}\right) \\ & + (\omega+2) \sum\nolimits_{i=1}^{n} \log x_i - \frac{\theta}{2} \sum\nolimits_{i=1}^{n} x_{i^2} \end{aligned} \tag{9}$$

After, differentiating the log likelihood w.r.to θ, and equate to zero, we have:

$$\hat{\theta}_{\text{mle}} = \frac{n\omega + 3n}{\sum_{i=1}^{n} x_{i^2}} \tag{10}$$

2.2 Bayesian Estimation of Weighted Boltzmann Maxwell Distribution using different loss functions

2.2.1 Estimation using extension of Jeffery's prior

The Joint Probability Density Function of θ and x is given by:

$$f_1(x, \theta) = \frac{\theta^{\frac{n\omega+3n}{2} - 2c_1} \sum {x_i}^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\omega+3}{2}\right)^n} \exp\left(-\frac{\theta}{2} \sum x_{i^2}\right) \tag{11}$$

And, the marginal distribution function of θ and x is given by:

$$f_1(x) = \frac{\sum {x_i}^{\omega+2}}{\Gamma\left(\frac{\omega+3}{2}\right)^n} \frac{\Gamma\left(\frac{n\omega+3n-4c_1+2}{2}\right) 2^{\frac{2n-4c_1+2}{2}}}{\left(\sum x_{i^2}\right)^{\frac{n\omega+3n-4c_1+2}{2}}} \tag{12}$$

Substituting the above two Eqs. (11) and (12), we get the Posterior Probability Density function of θ and x is given by:

$$\pi_1(\theta/x) = \frac{\theta^{\frac{n\omega+3n-4c_1}{2}}\left(\frac{\sum x_{i^2}}{2}\right)^{\frac{n\omega+3n-4c_1+2}{2}}}{\Gamma\left(\frac{n\omega+3n-4c_1+2}{2}\right)} \exp\left(-\frac{\theta}{2}\sum x_{i^2}\right) \tag{13}$$

2.2.1.1 Bayes' estimator under squared error loss function

The Risk Function Under SELF is given as:

$$R_{(\mathrm{Sq,EJ})}\left(\hat{\theta}\right) = c\hat{\theta}^2 + \frac{4c\left(\frac{n\omega+3n-4c_1+4}{2}\right)\left(\frac{n\omega+3n-4c_1+2}{2}\right)}{\left(\sum x_{i^2}\right)^2.} - \frac{4c\hat{\theta}\left(\frac{n\omega+3n-4c_1+2}{2}\right)}{.\left(\sum x_{i^2}\right)} \tag{14}$$

After solving the above risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Sq,EJ})} = \frac{n\omega+3n-4c_1+2}{\left(\sum x_{i^2}\right)} \tag{15}$$

2.2.1.2 Baye's estimator under precautionary: Loss function

The Risk Function Under Precautionary Loss Function is given as:

$$R_{(\mathrm{pr,EJ})}\left(\hat{\theta}\right) = c\hat{\theta} + \frac{4c\left(\frac{n\omega+3n-4c_1+4}{2}\right)\left(\frac{n\omega+3n-4c_1+2}{2}\right)}{\hat{\theta}\left(\sum x_{i^2}\right)^2.} - \frac{4c\left(\frac{n\omega+3n-4c_1+2}{2}\right)}{\left(\sum x_{i^2}\right)} \tag{16}$$

After solving the above risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{pr,Ej})} = \frac{\sqrt{(n\omega+3n-4c_1+4)(n\omega+3n-4c_1+2)}}{\left(\sum x_{i^2}\right)^2} \tag{17}$$

2.2.1.3 Baye's estimator under the Al-Bayyati's loss function

The Risk function under Al-Bayyati's Loss Function is given as:

$$R_{(\mathrm{Al,EJ})}\left(\hat{\theta}\right) = \frac{2^{c_2}\hat{\theta}^2\,\Gamma\left(\frac{n\omega+3n-4c_1+2c_2+2}{2}\right)}{\Gamma\left(\frac{n\omega+3n-4c_1+2}{2}\right)\left(\sum x_{i^2}\right)^{c_2}.} + \frac{2^{(c_2+2)}\Gamma\left(\frac{n\omega+3n-4c_1+2c_2+6}{2}\right)}{\Gamma\left(\frac{n\omega+3n-4c_1+2}{2}\right)\left(\sum x_{i^2}\right)^{(c_2+2)}}$$
$$-\frac{2^{(c_2+2)}\hat{\theta}\,\Gamma\left(\frac{n\omega+3n-4c_1+2c_2+4}{2}\right)}{\Gamma\left(\frac{n\omega+3n-4c_1+2}{2}\right)\left(\sum x_{i^2}\right)^{(c_2+1)}} \tag{18}$$

After solving the above risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Al,EJ})} = \frac{(n\omega+3n-4c_1+2c_2+2)}{\left(\sum x_{i^2}\right)} \tag{19}$$

2.2.1.4 Baye's estimator under the combination of Stein's loss function

The Risk function under the Stein's Loss function is given as:

$$R_{(\mathrm{St,Ej})}\left(\hat{\theta}\right) = \frac{\hat{\theta}\left(\frac{\sum x_{i^2}}{2}\right)}{\frac{n\omega+3n-4c_1}{2}} - \left(\ \log\ \hat{\theta}\ \right) + e^{-t} - 1 \tag{20}$$

After solving the above risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{St,Ej})} = \frac{n\omega + 3n - 4c_1}{\left(\sum x_{i^2}\right)} \tag{21}$$

2.2.2 Bayesian estimation under gamma (α, β) prior distributions

The Joint Probability Density Function of Maxwell-Boltzmann Distribution Using Gamma Prior Distribution is given as:

$$f_2(x,\theta) = \frac{\theta^{\left(\frac{n\omega+3n}{2}\right)} \sum x_i^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\hat{\omega}+3}{2}\right)^n} \exp\left(-\frac{\theta}{2}\sum x_{i^2}\right) \frac{\beta^\alpha}{\Gamma(\alpha)}\ \exp\left(-\beta\theta\right) \theta^{(\alpha-1)} \tag{22}$$

Also, the Marginal density function of x is given by:

$$f_2(x) = \frac{\sum x_i^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\omega+3}{2}\right)^n} \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2} + \beta\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}} \tag{23}$$

Using above Two Results (22) and (23), we get the posterior Probability Density Function:

$$\pi_2(\theta/x) = \frac{\theta^{\frac{n\omega+3n+2\alpha-2}{2}} \left(\frac{\sum x_{i^2}}{2} + \beta\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)} \exp\ \left(-\theta\left(\frac{\sum x_{i^2}}{2} + \beta\right)\right) \tag{24}$$

2.2.2.1 Under squared-error loss function

The Risk function under Squared Error Function is given as:

$$R_{(\mathrm{Sq,gp})}\left(\hat{\theta}\right) = c\hat{\theta}^2 + \frac{c\ \left(\frac{n\omega+3n+2\alpha+2}{2}\right)\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2} + \beta\right)^2} - \frac{2c\hat{\theta}\ \left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2} + \beta\right)} \tag{25}$$

After solving the above Risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Sq,gp})} = \frac{n\omega + 3n + 2\alpha}{\sum x_{i^2} + 2\beta} \tag{26}$$

2.2.2.2 Under precautionary loss function

The Risk function under precautionary Loss function is given as:

$$R_{(\mathrm{Pr,gp})}\left(\hat{\theta}\right)=c\hat{\theta}+c\hat{\theta}^{-1}\frac{\left(\frac{n\omega+3n+2\alpha+2}{2}\right)\left(\frac{n\omega+3n+2\alpha}{2}\right)\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{\left(\frac{4}{2}\right)}}-2c\frac{\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{\left(\frac{2}{2}\right)}} \tag{27}$$

After solving the above Risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Pr,gp})}=\frac{\sqrt{(n\omega+3n+2\alpha+2)(n\omega+3n+2\alpha)}}{\left(\sum x_{i^2}+2\beta\right)} \tag{28}$$

2.2.2.3 Under Al-Bayyati's loss function

The Risk function under Al-Bayyati's Loss function:

$$R_{(\mathrm{Al,gp})}\left(\hat{\theta}\right)=\frac{\hat{\theta}^2}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}\frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{(c_2)}}+\frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2+4}{2}\right)}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{(c_2+2)}}$$
$$-\frac{2\hat{\theta}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}\frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2+2}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{(c_2+1)}} \tag{29}$$

After solving $\frac{\partial R_{(\mathrm{Al,gp})}\left(\hat{\theta}\right)}{\partial\hat{\theta}}=0$ for $\hat{\theta}$, we will have the Baye's estimator given by:

$$\hat{\theta}_{\mathrm{Al,gp})}=\frac{n\omega+3n+2\alpha+2c_2}{\sum x_{i^2}+2\beta} \tag{30}$$

2.2.2.4 Under Stein's loss function

The Risk function under Stein Loss Function is given by:

$$R_{(\mathrm{St,gp})}\left(\hat{\theta}\right)=\int_0^\infty\left(\frac{\hat{\theta}}{\theta}-\log\frac{\hat{\theta}}{\theta}-1\right)\frac{\theta^{\frac{n\omega+3n+2\alpha-2}{2}}\left(\frac{\sum x_{i^2}}{2}+\beta\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}\exp\left(-\theta\left(\frac{\sum x_{i^2}}{2}+\beta\right)\right)d\theta$$

$$R_{(\mathrm{St,gp})}\left(\hat{\theta}\right)=\hat{\theta}\frac{\Gamma\left(\frac{n\omega+3n+2\alpha-2}{2}\right)\left(\frac{\sum x_{i^2}}{2}+\beta\right)}{\left(\frac{n\omega+3n+2\alpha-2}{2}\right)\Gamma\left(\frac{n\omega+3n+2\alpha-2}{2}\right)}-\log\left(\hat{\theta}\right)+e^{-t}-1 \tag{31}$$

After solving the above Risk function, we will have required Baye's estimator:

$$\hat{\theta}_{(\mathrm{St,gp})}=\frac{n\omega+3n+2\alpha-2}{\left(\sum x_{i^2}+2\beta\right)} \tag{32}$$

2.2.3 Bayesian estimation under exponential (α) prior distributions

The Joint Probability Density Function of Weighted Maxwell-Boltzmann Distribution Using Exponential Prior Distribution is given as:

$$f_2(x,\theta) = \frac{\theta^{\left(\frac{n\omega+3n}{2}\right)} \sum {x_i}^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\hat{\omega}+3}{2}\right)^n} \exp\left(-\frac{\theta}{2}\sum x_{i^2}\right) \frac{1}{\Gamma(\alpha)} \exp(-\theta)\, \theta^{(\alpha-1)} \tag{33}$$

Also, the Marginal density function of x is given by:

$$f_2(x) = \frac{\sum {x_i}^{\omega+2}}{2^{n\left(\frac{\omega+1}{2}\right)} \Gamma\left(\frac{\omega+3}{2}\right)^n} \frac{1}{\Gamma(\alpha)} \frac{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}} \tag{34}$$

Using above Two Results (33) and (34), we get the posterior Probability Density Function:

$$\pi_2(\theta/x) = \frac{\theta^{\frac{n\omega+3n+2\alpha-2}{2}} \left(\frac{\sum x_{i^2}}{2}+1\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)} \exp\left(-\theta\left(\frac{\sum x_{i^2}}{2}+1\right)\right) \tag{35}$$

2.2.3.1 Under squared-error loss function

The Risk function under Squared Error Function is given as:

$$R_{(\mathrm{Sq,Ep})}\left(\hat{\theta}\right) = c\hat{\theta}^2 + \frac{c\left(\frac{n\omega+3n+2\alpha+2}{2}\right)\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)^2} - \frac{2c\hat{\theta}\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)} \tag{36}$$

After solving the above Risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Sq,ep})} = \frac{n\omega+3n+2\alpha}{\sum x_{i^2}+2} \tag{37}$$

2.2.3.2 Under precautionary loss function

The Risk function under precautionary Loss function is given as:

$$R_{(\mathrm{Pr,ep})}\left(\hat{\theta}\right) = c\hat{\theta} + c\hat{\theta}^{-1} \frac{\left(\frac{n\omega+3n+2\alpha+2}{2}\right)\left(\frac{n\omega+3n+2\alpha}{2}\right)\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)\left(\frac{\sum x_{i^2}}{2}+1\right)^{\left(\frac{4}{2}\right)}} - 2c\frac{\left(\frac{n\omega+3n+2\alpha}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)^{\left(\frac{2}{2}\right)}}$$

After solving the above Risk function, we get the Baye's estimator:

$$\hat{\theta}_{(\mathrm{Pr,ep})} = \frac{\sqrt{(n\omega+3n+2\alpha+2)(n\omega+3n+2\alpha)}}{\left(\sum x_{i^2}+2\right)} \tag{38}$$

2.2.3.3 Under Al-Bayyati's loss function

The Risk function under Al-Bayyati's Loss function:

$$R_{(\mathrm{Al,ep})}(\hat{\theta}) = \frac{\hat{\theta}^2}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)} \frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)^{(c_2)}} + \frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2+4}{2}\right)}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)\left(\frac{\sum x_{i^2}}{2}+1\right)^{(c_2+2)}} - \frac{2\hat{\theta}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)} \frac{\Gamma\left(\frac{n\omega+3n+2\alpha+2c_2+2}{2}\right)}{\left(\frac{\sum x_{i^2}}{2}+1\right)^{(c_2+1)}} \tag{39}$$

After solving the above Risk Function, we will have the Baye's estimator given by:

$$\hat{\theta}_{\mathrm{Al,Ep})} = \frac{n\omega+3n+2\alpha+2c_2}{\sum x_{i^2}+2} \tag{40}$$

2.2.3.4 Under Stein's loss function

The Risk function under Stein Loss Function is given by:

$$R_{(\mathrm{St,ep})}(\hat{\theta}) = \int_0^\infty \left(\frac{\hat{\theta}}{\theta} - \log\frac{\hat{\theta}}{\theta} - 1\right) \frac{\theta^{\frac{n\omega+3n+2\alpha-2}{2}}\left(\frac{\sum x_{i^2}}{2}+1\right)^{\left(\frac{n\omega+3n+2\alpha}{2}\right)}}{\Gamma\left(\frac{n\omega+3n+2\alpha}{2}\right)} \exp\left(-\theta\left(\frac{\sum x_{i^2}}{2}+1\right)\right)d\theta$$

$$R_{(\mathrm{St,ep})}(\hat{\theta}) = \hat{\theta}\,\frac{\Gamma\left(\frac{n\omega+3n+2\alpha-2}{2}\right)\left(\frac{\sum x_{i^2}}{2}+1\right)}{\left(\frac{n\omega+3n+2\alpha-2}{2}\right)\Gamma\left(\frac{n\omega+3n+2\alpha-2}{2}\right)} - \log(\hat{\theta}) + e^{-t} - 1 \tag{41}$$

After solving the above Risk function, we will have required Baye's estimator:

$$\hat{\theta}_{(\mathrm{St,ep})} = \frac{n\omega+3n+2\alpha-2}{\left(\sum x_{i^2}+2\right)} \tag{42}$$

3. Simulation study of weighted Maxwell-Boltzmann distribution

In this section, we conduct the simulation studies of weighted Maxwell-Boltzmann distribution to examine the performance of the MLEs and Bayesian estimators under different prior's like extension of Jeffrey's' prior, Gamma prior and Exponential prior under different loss functions in terms of expected estimates, biases, variances and mean squared errors by considering different parameter combinations. For the simulation study, sample size is taken as n = (25, 100, 300) to observe the effect of small, moderate and large samples on the estimators. We have conducted the simulation procedure for different random parameter combinations and the process was repeated 2000 times. From the simulation results, it is concluded that the performances of the Bayesian and MLEs become better when the sample size increases. In terms of MSE, the Bayesian estimators under Gamma prior perform better (see **Table 1**). In specific, from **Table 2**, extension of Jeffery's prior under Al-Bayyati's error loss function and stein's loss function gives smaller MSE's as compared to other loss functions.

From **Table 1**, we can see that the performances of the Bayesian and MLEs become better when the sample size increases. For large samples, Gamma prior

n	θ	ω	α	β	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
25	0.5	0.5	0.5	0.5	0.5	$E(\theta)$	0.504729	0.527842	0.526203	0.511548	0.532156
						Bias	0.004729	0.027842	0.026203	0.011548	0.032156
						Variance	0.003753	0.007177	0.006050	0.003911	0.007792
						MSE	0.003775	0.007952	0.006737	0.004044	0.008826
100	0.5	0.5	0.5	0.5	0.5	$E(\theta)$	0.506417	0.501861	0.502694	0.504235	0.500950
						Bias	0.006417	0.001861	0.002694	0.004235	0.000950
						Variance	0.001238	0.001789	0.001571	0.001552	0.001159
						MSE	0.001279	0.001792	0.001578	0.001570	0.001160
300	0.5	0.5	0.5	0.5	0.5	$E(\theta)$	0.500341	0.501755	0.500753	0.506296	0.501444
						Bias	0.000341	0.001755	0.000753	0.006296	0.001444
						Variance	0.000434	0.000593	0.000357	0.000464	0.000416
						MSE	0.000434	0.000596	0.000358	0.000503	0.000418
25	1.5	0.5	0.4	0.5	0.5	$E(\theta)$	1.555484	1.497774	1.547039	1.552606	1.478335
						Bias	0.055484	−0.002226	0.047039	0.052606	−0.021665
						Variance	0.059091	0.055145	0.057945	0.054047	0.042449
						MSE	0.062169	0.055150	0.060158	0.056815	0.042918
100	1.5	0.5	0.4	0.5	0.5	$E(\theta)$	1.511161	1.487596	1.479585	1.506022	1.510950
						Bias	0.011161	−0.012404	−0.020415	0.006022	0.010950
						Variance	0.010559	0.012894	0.013387	0.012110	0.011800
						MSE	0.010683	0.013048	0.013804	0.012146	0.011919

n	θ	ω	α	β	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
300	1.5	0.5	0.4	0.5	0.5	$E(\theta)$	1.500168	1.510651	1.493056	1.514740	1.502537
						Bias	0.000168	0.010651	−0.006944	0.014740	0.002537
						Variance	0.005194	0.003724	0.004315	0.005630	0.004500
						MSE	0.005194	0.003838	0.004363	0.005847	0.004506
25	2.0	1.6	0.4	1.5	1.5	$E(\theta)$	1.938975	1.991177	2.000771	1.927722	1.956042
						Bias	−0.061025	−0.008823	0.000771	−0.072278	−0.043958
						Variance	0.059838	0.070756	0.103288	0.065984	0.061025
						MSE	0.063562	0.070833	0.103288	0.071208	0.062957
100	2.0	1.6	0.4	1.5	1.5	$E(\theta)$	1.969832	1.963989	1.977604	2.015424	1.975888
						Bias	−0.030168	−0.036011	−0.022396	0.015424	−0.024112
						Variance	0.016741	0.017403	0.012035	0.016688	0.014520
						MSE	0.017651	0.018700	0.012537	0.016926	0.015102
300	2.0	1.6	0.4	1.5	1.5	$E(\theta)$	2.004078	1.987208	1.994396	2.003804	2.002971
						Bias	0.004078	−0.012792	−0.005604	0.003804	0.002971
						Variance	0.006379	0.006670	0.005228	0.005783	0.008284
						MSE	0.006396	0.006834	0.005260	0.005798	0.008293
25	2.5	1.6	0.4	1.5	2.0	$E(\theta)$	2.394049	2.404621	2.347984	2.500847	2.458514
						Bias	−0.105951	−0.095379	−0.152016	0.000847	−0.041486
						Variance	0.098064	0.118924	0.119786	0.154811	0.094105
						MSE	0.109290	0.128021	0.142895	0.154812	0.095827

n	θ	ω	α	β	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
100	2.5	1.6	0.4	1.5	2.0	$E(\theta)$	2.515576	2.494660	2.491992	2.484427	2.508391
						Bias	0.015576	−0.005340	−0.008008	−0.015573	0.008391
						Variance	0.022966	0.026813	0.021573	0.023537	0.023719
						MSE	0.023209	0.026842	0.021637	0.023780	0.023790
300	2.5	1.6	0.4	1.5	2.0	$E(\theta)$	2.505136	2.490348	2.502325	2.488103	2.519407
						Bias	0.005136	−0.009652	0.002325	−0.011897	0.019407
						Variance	0.008742	0.008224	0.006055	0.010543	0.009009
						MSE	0.008768	0.008317	0.006060	0.010684	0.009386

ml, *maximum likelihood;* sq, *squared error loss function;* pre, *precautionary loss function;* alb, *Al-Bayyati's loss function;* ste, *Stein's loss function.*

Table 1.
Average estimate, bias, variance and mean squared error for $(\hat{\theta})$ *under gamma prior.*

n	θ	ω	c_1	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
25	0.5	0.5	0.5	0.5	$E(\theta)$	0.504550	0.508841	0.519292	0.520839	0.525694
					Bias	0.004550	0.008841	0.019292	0.020839	0.025694
					Variance	0.004979	0.005660	0.008970	0.006678	0.009465
					MSE	0.004999	0.005738	0.009342	0.007112	0.010125
100	0.5	0.5	0.5	0.5	$E(\theta)$	0.505610	0.508184	0.501928	0.492615	0.501255
					Bias	0.005610	0.008184	0.001928	−0.007385	0.001255
					Variance	0.001313	0.001677	0.001707	0.001318	0.001377
					MSE	0.001344	0.001744	0.001710	0.001372	0.001379
300	0.5	0.5	0.5	0.5	$E(\theta)$	0.495559	0.503048	0.503241	0.500495	0.504760
					Bias	−0.004441	0.003048	0.003241	0.000495	0.004760
					Variance	0.000391	0.000500	0.000467	0.000532	0.000487
					MSE	0.000411	0.000510	0.000478	0.000533	0.000510
25	1.6	0.5	0.5	0.5	$E(\theta)$	1.656361	1.652441	1.590889	1.596342	1.624334
					Bias	0.056361	0.052441	−0.009111	−0.003658	0.024334
					Variance	0.072186	0.080721	0.048146	0.046070	0.052856
					MSE	0.075363	0.083471	0.048229	0.046084	0.053448
100	1.6	0.5	0.5	0.5	$E(\theta)$	1.584041	1.592563	1.627598	1.617623	1.621388
					Bias	−0.015959	−0.007437	0.027598	0.017623	0.021388
					Variance	0.013766	0.013701	0.012200	0.011394	0.014179
					MSE	0.014021	0.013756	0.012961	0.011704	0.014636
300	1.6	0.5	0.5	0.5	$E(\theta)$	1.595942	1.621590	1.615841	1.605064	1.600224
					Bias	−0.004058	0.021590	0.015841	0.005064	0.000224
					Variance	0.006184	0.006401	0.005696	0.003994	0.005462
					MSE	0.006200	0.006867	0.005947	0.004020	0.005462
25	2.5	1.0	1.5	0.5	$E(\theta)$	2.479248	2.477210	2.416603	2.493049	2.428281
					Bias	−0.020752	−0.022790	−0.083397	−0.006951	−0.071719
					Variance	0.128038	0.127245	0.113067	0.149621	0.111466
					MSE	0.128468	0.127765	0.120022	0.149670	0.116610
100	2.5	1.0	1.5	0.5	$E(\theta)$	2.498178	2.502858	2.527984	2.482989	2.488065
					Bias	−0.001822	0.002858	0.027984	−0.017011	−0.011935
					Variance	0.037896	0.029747	0.030859	0.020501	0.023293
					MSE	0.037899	0.029756	0.031642	0.020791	0.023436
300	2.5	1.0	1.5	0.5	$E(\theta)$	2.510585	2.500611	2.490367	2.507471	2.477393
					Bias	0.010585	0.000611	−0.009633	0.007471	−0.022607
					Variance	0.010037	0.009817	0.007991	0.008569	0.012196
					MSE	0.010149	0.009818	0.008083	0.008625	0.012707
25	2.5	1.0	0.5	1.5	$E(\theta)$	2.566472	2.690959	2.607021	2.559121	2.582644
					Bias	0.066472	0.190959	0.107021	0.059121	0.082644
					Variance	0.133694	0.132587	0.132802	0.133658	0.156276
					MSE	0.138112	0.169052	0.144256	0.137153	0.163106

n	θ	ω	c_1	c_2	Criterion	$\hat{\theta}_{\text{ml}}$	$\hat{\theta}_{\text{sq}}$	θ_{pre}	$\hat{\theta}_{\text{alb}}$	θ_{ste}
100	2.5	1.0	0.5	1.5	$E(\theta)$	2.505612	2.535815	2.524166	2.489736	2.514349
					Bias	0.005612	0.035815	0.024166	−0.010264	0.014349
					Variance	0.042607	0.029863	0.029906	0.031724	0.032759
					MSE	0.042639	0.031146	0.030490	0.031829	0.032965
300	2.5	1.0	0.5	1.5	$E(\theta)$	2.499279	2.508299	2.510490	2.490229	2.487861
					Bias	−0.000721	0.008299	0.010490	−0.009771	−0.012139
					Variance	0.011396	0.009322	0.011898	0.011426	0.011835
					MSE	0.011397	0.009391	0.012008	0.011522	0.011982

ml, *maximum likelihood;* sq, *squared error loss function;* pre, *precautionary loss function;* alb, *Al-Bayyati's loss function;* ste, *Stein's loss function.*

Table 2.
Average estimate, bias, variance and mean squared error for $(\hat{\theta})$ *under extension of Jeffery's prior.*

under squared error loss function and Al-Bayyati's loss function gives smaller MSE's as compared to other loss functions and MLEs.

From **Table 3**, we can see that the performances of the Bayesian and MLEs become better when the sample size increases. Exponential prior under squared error loss function and stein's loss function gives smaller MSE's as compared to other loss functions. Thus, Exponential prior under squared error loss function and stein's loss function can be preferred for parameter estimation.

4. Applications of weighted Maxwell-Boltzmann distribution

In this section, we present the goodness of fit of weighted Maxwell-Boltzmann distribution (WMB). For testing the goodness of fit of weighted Maxwell-Boltzmann distribution over Maxwell-Boltzmann (MB), length biased Maxwell-Boltzmann (LBMB) and area biased Maxwell-Boltzmann (ABMB) distributions, following two data sets have been considered.

Data set I is regarding tensile strength, measured in GPA, of 69 carbon fibers tested under tension at gauge lengths of 20 mm, Bader and Priest [29].

From **Table 4**, it has been observed that weighted Maxwell-Boltzmann distribution have the lesser AIC, AICC, $-\log L$ and BIC values as compared to Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions. Hence we can conclude that the Weighted Maxwell-Boltzmann distribution leads to a better fit than the Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions in case of analyzing the data set I.

Data set II is regarding the strength data and it represents the strength measured in GPA for single carbon fibers and impregnated 1000-carbon fiber tows. Single fibers were tested under tension at gauge lengths of 10 mm with sample sizes n = 63; see Bader and Priest [29] and Surles and Padgett [30].

From **Table 5**, it has been observed that weighted Maxwell-Boltzmann distribution have the lesser AIC, AICC, $-\log L$ and BIC values as compared to Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions. Hence we can conclude that the Weighted Maxwell-Boltzmann distribution leads to a better fit than the Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions in case of analyzing the data set II.

n	θ	ω	α	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
25	0.5	0.5	0.5	0.5	$E(\theta)$	0.489954	0.518542	0.508290	0.497323	0.494616
					Bias	−0.010046	0.018542	0.008290	−0.002677	−0.005384
					Variance	0.006497	0.005296	0.005568	0.005993	0.005576
					MSE	0.006598	0.005640	0.005636	0.006000	0.005605
100	0.5	0.5	0.5	0.5	$E(\theta)$	0.501216	0.510728	0.499014	0.503758	0.508104
					Bias	0.001216	0.010728	−0.000986	0.003758	0.008104
					Variance	0.000940	0.001242	0.001713	0.001204	0.001588
					MSE	0.000941	0.001357	0.001714	0.001218	0.001654
300	0.5	0.5	0.5	0.5	$E(\theta)$	0.500133	0.496843	0.497977	0.499664	0.499775
					Bias	0.000133	−0.003157	−0.002023	−0.000336	−0.000225
					Variance	0.000413	0.000407	0.000447	0.000357	0.000340
					MSE	0.000413	0.000417	0.000451	0.000357	0.000340
25	1.5	0.5	0.4	0.5	$E(\theta)$	1.483379	1.482564	1.511903	1.540451	1.566552
					Bias	−0.016621	−0.017436	0.011903	0.040451	0.066552
					Variance	0.066483	0.048448	0.049620	0.060298	0.047485
					MSE	0.066759	0.048752	0.049761	0.061934	0.051914
100	1.5	0.5	0.4	0.5	$E(\theta)$	1.490401	1.508552	1.501250	1.501611	1.472398
					Bias	−0.009599	0.008552	0.001250	0.001611	−0.027602
					Variance	0.013307	0.012242	0.014316	0.015729	0.012469
					MSE	0.013399	0.012316	0.014317	0.015731	0.013231

n	θ	ω	α	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
300	1.5	0.5	0.4	0.5	$E(\theta)$	1.517577	1.511777	1.521694	1.499300	1.505245
					Bias	0.017577	0.011777	0.021694	−0.000700	0.005245
					Variance	0.004328	0.004223	0.003876	0.003398	0.003845
					MSE	0.004637	0.004362	0.004346	0.003398	0.003873
25	2.0	1.6	0.4	1.5	$E(\theta)$	2.044911	2.009608	1.985619	1.980819	1.993491
					Bias	0.044911	0.009608	−0.014381	−0.019181	−0.006509
					Variance	0.112842	0.062620	0.057835	0.085826	0.071638
					MSE	0.114859	0.062712	0.058042	0.086194	0.071680
100	2.0	1.6	0.4	1.5	$E(\theta)$	1.996708	1.997635	2.008162	2.021764	2.002779
					Bias	−0.003292	−0.002365	0.008162	0.021764	0.002779
					Variance	0.015238	0.012785	0.013466	0.014708	0.015945
					MSE	0.015248	0.012791	0.013533	0.015181	0.015953
300	2.0	1.6	0.4	1.5	$E(\theta)$	1.997046	2.002154	1.992272	2.005303	2.004678
					Bias	−0.002954	0.002154	−0.007728	0.005303	0.004678
					Variance	0.006380	0.005365	0.005987	0.006364	0.006716
					MSE	0.006389	0.005370	0.006046	0.006392	0.006738
25	2.5	1.6	0.4	2.0	$E(\theta)$	2.471114	2.476397	2.496794	2.507617	2.539850
					Bias	−0.028886	−0.023603	−0.003206	0.007617	0.039850
					Variance	0.105051	0.096227	0.163694	0.108075	0.135941
					MSE	0.105885	0.096785	0.163704	0.108134	0.137529

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n	θ	ω	α	c_2	Criterion	$\hat{\theta}_{ml}$	$\hat{\theta}_{sq}$	θ_{pre}	$\hat{\theta}_{alb}$	θ_{ste}
100	2.5	1.6	0.4	2.0	$E(\theta)$	2.507110	2.499740	2.498220	2.517427	2.508359
					Bias	0.007110	−0.000260	−0.001780	0.017427	0.008359
					Variance	0.024000	0.027471	0.033577	0.031666	0.028237
					MSE	0.024051	0.027471	0.033581	0.031970	0.028307
300	2.5	1.6	0.4	2.0	$E(\theta)$	2.515885	2.503115	2.476479	2.507093	2.504668
					Bias	0.015885	0.003115	−0.023521	0.007093	0.004668
					Variance	0.011382	0.007855	0.011195	0.009851	0.007705
					MSE	0.011635	0.007865	0.011749	0.009901	0.007727

ml, *maximum likelihood;* sq, *squared error loss function;* pre, *precautionary loss function;* alb, *Al-Bayyati's loss function;* ste, *Stein's loss function.*

Table 3.
Average estimate, bias, variance and mean squared error for $(\hat{\theta})$ *under exponential prior.*

Distribution	α_{ml}	θ_{ml}	$-2\log\ (l)$	AIC	BIC	AICC
WMB	9.079	1.923	50.393	104.787	109.255	104.968
MB	0	0.478	74.633	151.265	153.499	151.325
LBMB	1	0.637	66.713	135.426	137.660	135.485
ABMB	2	0.796	61.385	124.770	127.004	124.829

Table 4.
*Model comparison using AIC, AICC, BIC and -log*L *criterion for data set 1.*

Distribution	α_{ml}	θ_{ml}	$-2\log\ (l)$	AIC	BIC	AICC
WMB	9.971	1.332	57.656	119.311	123.598	119.511
MB	0	0.308	81.585	165.170	167.313	165.235
LBMB	1	0.411	74.165	150.330	152.473	150.395
ABMB	2	0.513	69.111	140.222	142.366	140.288

Table 5.
*Model comparison using AIC, AICC, BIC and -log*L *criterion for data set II.*

5. Conclusions

1. From the simulation Study, it was observed that the performances of the Bayesian and MLEs become better, when the sample size increases.

2. In terms of MSE, the Bayesian estimators under Gamma prior perform better. In specific, extension of Jeffery's prior under Al-Bayyati's error loss function and stein's loss function gives smaller MSE's as compared to other loss functions.

3. For large samples, Gamma prior under squared error loss function and Al-Bayyati's loss function gives smaller MSE's as compared to other loss functions and MLEs. Exponential prior under squared error loss function and stein's loss function gives smaller MSE's as compared to other loss functions.

4. Thus, Exponential prior under squared error loss function and stein's loss function can be preferred for parameter estimation.

5. It has been observed that weighted Maxwell-Boltzmann distribution have the lesser AIC, AICC, $-\log L$ and BIC values as compared to Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions. Hence we can conclude that the Weighted Maxwell-Boltzmann distribution leads to a better fit than the Maxwell-Boltzmann, length biased Maxwell-Boltzmann and area biased Maxwell-Boltzmann distributions in case of analyzing the data set I and II.

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